

National Exams December 2015  
04-BS-1, Mathematics  
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.
  2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
  3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
  4. All questions are of equal value.
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Marking Scheme:

1. (a) 10 marks, (b) 10 marks
2. (a) 10 marks, (b) 10 marks
3. (a) 10 marks, (b) 5 marks, (c) 5 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. 20 marks
8. 20 marks

1. Find the general solutions of the following differential equations:

(a)  $y' + xy = 2xe^{-x^2}$ ,

(b)  $y'' + y' - 6y = 0$ .

Note that in each case, ' denotes differentiation with respect to  $x$ .

2. Solve the following initial value problems:

(a)  $y' + 2ty^2 = 0$ ,  $y(1) = 2$ ,

(b)  $y'' - 12y' + 45y = 18 \cos(3t)$ ,  $y(0) = 0$ ,  $y'(0) = 0$ .

Note that in each case, ' denotes differentiation with respect to  $t$ .

3. Consider the two lines defined as follows:

$$x = 3 + 2t, \quad y = 3, \quad z = 1 - t, \quad (\text{parameter } t);$$

$$x = s, \quad y = 1 - 2s, \quad z = 2 + s, \quad (\text{parameter } s).$$

- (a) Determine whether or not the two lines intersect, and if so, find the point of intersection.  
 (b) Find a third line orthogonal to both lines.  
 (c) Is there a plane containing both lines? If so, find an equation for that plane.

4. Find the maximum and minimum values of  $f(x, y, z) = x + y - 2z$  over the ellipse  $x^2 + y^2 + 4z^2 = 1$ .

5. Let  $S$  be the boundary of the region enclosed by the paraboloid  $z = 4 - x^2 - y^2$  and the plane  $z = 0$  and let

$$\mathbf{F}(x, y, z) = x^2\mathbf{i} - 2xy\mathbf{j} + x^2z\mathbf{k}.$$

Evaluate the surface integral  $\iint_S \mathbf{F} \cdot \mathbf{n} \, dA$ , where  $\mathbf{n}$  is the unit outward normal on  $S$ .

6. At what angle does the line represented parametrically by  $x = 1 - t$ ,  $y = t$ ,  $z = 2 + 3t$  intersect the hyperboloid  $z = 4 - x^2 + y^2$ ? You may leave your answer as an inverse sine or cosine.

7. Evaluate the line integral  $\oint_C \mathbf{v} \cdot d\mathbf{r}$  where  $C$  is the curve formed by the intersection of the cylinder  $x^2 + y^2 = 4$  and the plane  $z + 2x - y = 3$ , travelled counterclockwise as viewed from the positive  $z$ -axis, and  $\mathbf{v}$  is the vector function  $\mathbf{v} = x\mathbf{i} + (x - y)\mathbf{j} + yz\mathbf{k}$ .

8. Find the volume of the solid region inside the ellipsoid

$$x^2 + y^2 + 4z^2 = 5$$

and above the cone

$$z = \sqrt{x^2 + y^2}.$$