NATIONAL EXAMS
May 2014

Phys-A6: Solid State Physics

3 hours duration

NOTES:

1. If doubt exits as to the interpretation of any question, the candidate must submit with the answer paper, a clear statement of any assumption made.
2. Candidates may use one of two calculators, the Casio or Sharp approved models.
3. This is a CLOSED BOOK EXAM.
   Useful constants and equations have been annexed to the exam paper.
4. **Any FIVE (5) of the SEVEN (7) questions** constitute a complete exam paper.
   The first five questions as they appear in the answer book will be marked.
5. When answering questions, candidates must clearly indicate units for all parameters used or computed.

MARKING SCHEME

Questions	*Marks*				
1	(a) 3	(b) 5	(c) 8	(d) 4	
2	(a) 10	(b) 10			
3	(a) 12	(b) 8			
4	(a) 9	(b) 4	(c) 7		
5	(a) 2	(b) 2	(c) 4	(d) 8	(e) 4
6	(a) 4	(b) 4	(c) 12		
7	(a) 6	(b) 7	(c) 7		

1. The three Bravais lattices shown in Figure P1 all have the same width, height and depth.

3 pts (a) Name each lattice displayed in Figure P1.

5 pts (b) Calculate the *packing fraction* for the lattice of Figure P1c.
[Note: The volume of a sphere or radius r is $V = (4\pi r^3)/3$]

8 pts (c) Find the primitive translation vectors $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$ for the lattice of Figure P1b and find the primitive translation vectors $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$ for the corresponding *reciprocal lattice*.

4 pts (d) What are the Miller indices of a plane parallel to the grey area shown in Figure P1b that passes through the two bottom right hand side atoms (black spheres) of the lattice.

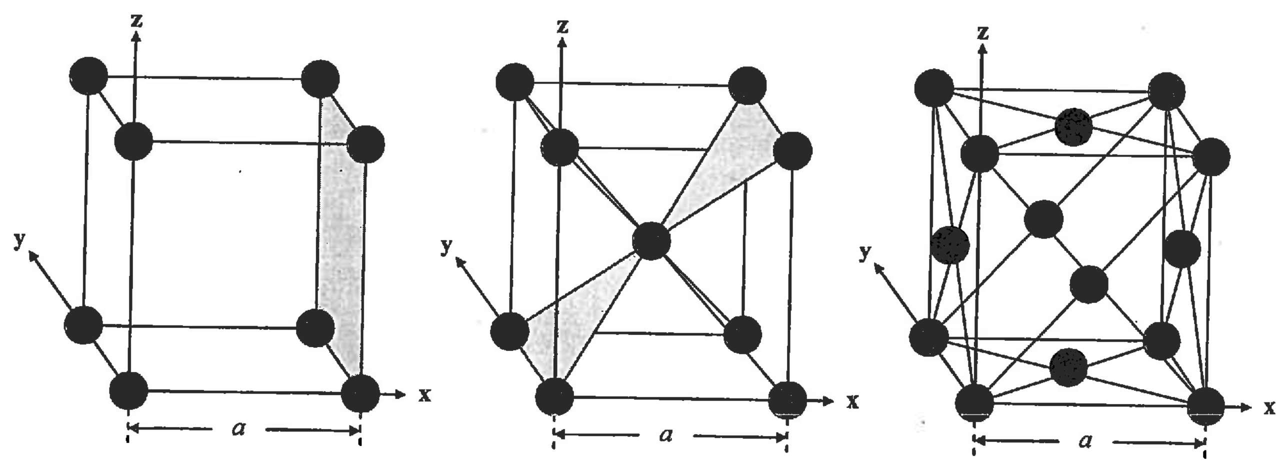


Figure P1a **Figure P1b** **Figure P1c**

2. The interaction between two inert gas atoms takes the form of the normalized Leonard-Jones potential $U(R)/\epsilon$ shown in Figure P2. Table T2 lists the properties of some inert gas crystals.

10 pts (a) Show that the minimum value of the curve occurs at $R/\sigma \approx 1.12$.

10 pts (b) Calculate the force between two adjacent atoms of Krypton (Kr) when they are 5 Å apart.

Table T2 - Properties of some inert gas

Inert gas	Distance to nearest neighbor (Å)	ϵ (10^{-16} erg)	σ (Å)
Ne	3.13	50	2.74
Ar	3.76	167	3.40
Kr	4.01	225	3.65
Xe	4.35	320	3.98

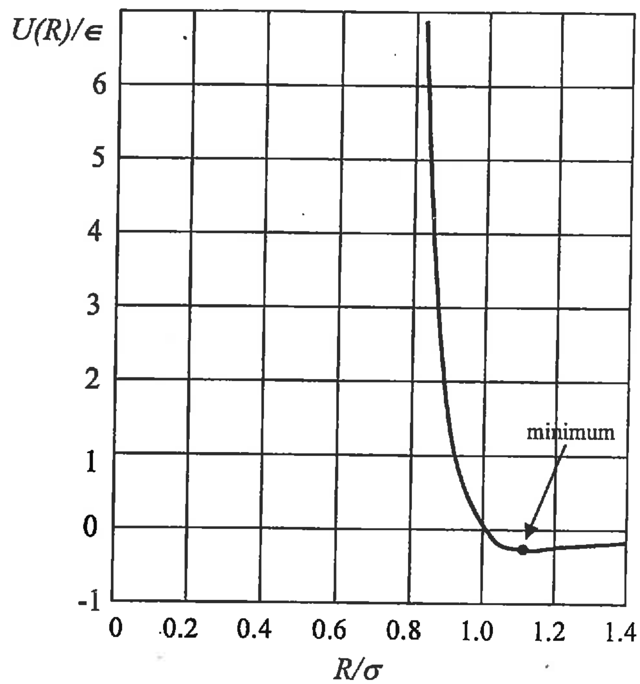


Figure P2

3. Consider longitudinal vibrations in a diatomic crystal with atoms of mass M_1 and M_2 connected with a force constant C between adjacent atoms. Undisplaced planes are illustrated in Figure P3a. Motion solutions are in the form of traveling waves with different amplitudes u and v on alternate planes. This leads to the following equations:

$$-\omega^2 M_1 u = Cv\left[1 + e^{-iKa}\right] - 2Cu \quad (1)$$

$$-\omega^2 M_2 v = Cu\left[1 + e^{+iKa}\right] - 2Cv \quad (2)$$

Solutions are possible only if

$$M_1 M_2 \omega^4 - 2C\,(M_1 + M_2)\omega^2 + 2C^2(1 - \cos Ka) = 0 \quad (3)$$

12 pts (a) The ω(K) response in the first Brillouin zone is shown in Figure P3b. Find expressions for optical phonon frequencies ω_1 and ω_2, and for acoustical phonon frequency ω_3 in terms of parameters C, M_1 and M_2.

8 pts (b) The vibration modes of a transverse wave for atoms carrying opposite charges are shown in Figure P3c. Briefly explain the origin of the terms "optical mode" and "acoustical mode".

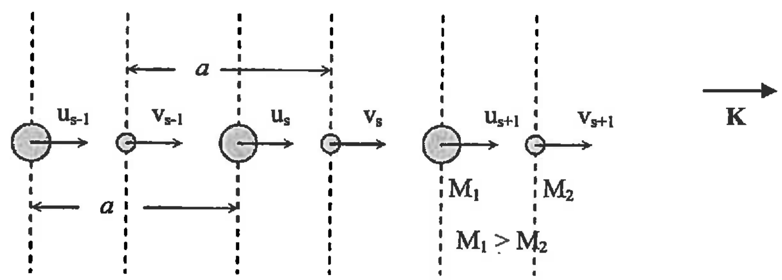
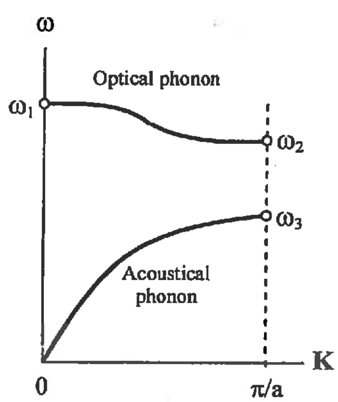


Figure P3a



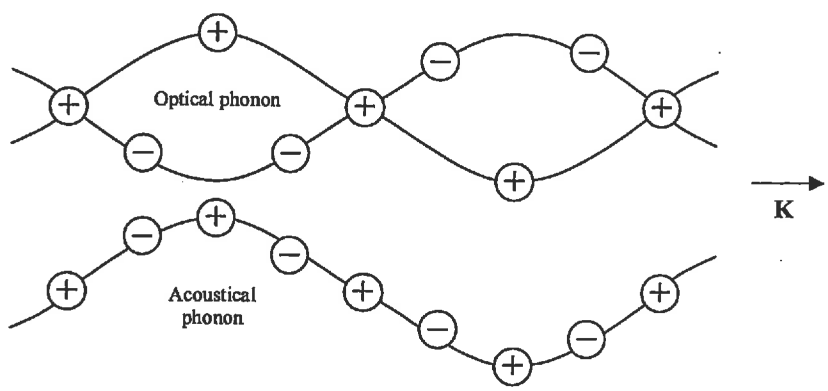


Figure P3b **Figure P3c**

4. Particles which behavior follows the Fermi-Dirac distribution are called *fermions*. The 3-dimensional Fermi surface of fermions is shown in Figure P4. Just like free electrons, the helium-3 (He^3) atoms behave like fermions. He^3 is composed of 3 atomic mass units: 2 protons and 1 neutron. The density of He^3 near absolute zero temperature is 0.081 g/cm^3.

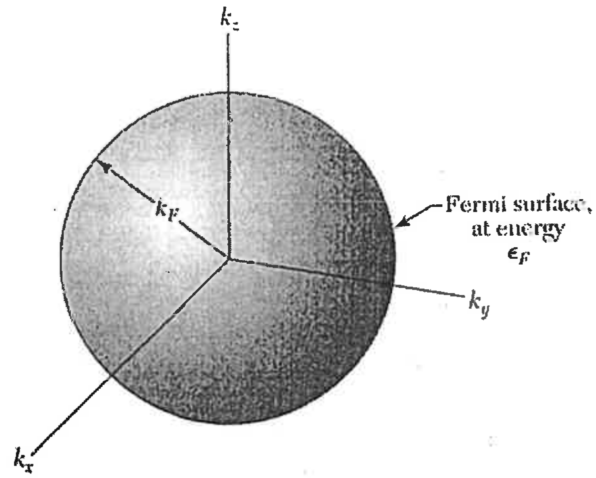


Figure P4

9 pts (a) Show that He^3 atoms have a Fermi energy ϵ_F of about 7 x 10^{-16} erg.

4 pts (b) What is the Fermi temperature T_F that corresponds to this Fermi energy?

7 pts (c) Assuming that the *chemical potential* is approximately equal to 1.5 ϵ_F at T= 45 °K, what is the probability that an atom of He^3 would occupy an energy level of $\epsilon = 2.5\ \epsilon_F$ at T= 45 °K?

5. Measurements for a device made of intrinsic Silicon (Si) have shown that the Si *band gap* is equal to E_g = 1.08 eV and that the electrons have an *effective mass* of 1.1*m* and the holes have an effective mass of 0.56*m* where *m* is the mass of an electron at rest.

2 pts (a) Briefly explain what is meant by *band gap*

2 pts (b) Briefly explain what is meant by *effective mass*

4 pts (c) Calculate the Fermi level of intrinsic Si at T = 300 °K.

8 pts (d) Calculate the hole concentration in the device if the temperature is T = 320 °K

4 pts (e) To improve the conductivity of the Si device, a high concentration of donor atoms are injected into the intrinsic Si. Briefly explain why such donor impurities improve conductivity; and, state what happens to the Fermi level.

6. The following questions refer to magnetism present or induced in crystal lattices.

4 pts (a) State what gives *paramagnetic* contributions to the magnetization of a substance.

4 pts (b) State what gives *diamagnetic* contributions to the magnetization of a substance.

12 pts (c) An inert gas has the following properties:

density: 0.214 g/cm^3
average atomic radius: 3.8 $x10^{-9}$ cm
number of neutrons: 2
number of protons: 2
number electrons: 2

It magnetic susceptibility is given by $\chi = -\frac{\mu_0 N Z e^2}{6m}\langle r^2 \rangle$

Evaluate the susceptibility of this inert gas in units of ***cm^3/mole*** and specify it the gas is *paramagnetic* or *diamagnetic*.

7. The following questions refer to the presence of point defects and dislocations in crystal lattices. Useful data regarding the diffusion of defects is provided in Table T7.

6 pts (a) Briefly explain the main difference between *elastic* deformations and *plastic* deformations of crystalline solids.

7 pts (b) If the energy to take an atom of Na from its normal lattice site to a lattice site at the surface of the crystal is 1.0 eV, calculate the required temperature T (in °K) to obtain a bulk defect concentration of 1 vacancy per 100000 atoms of Na.

7 pts (c) To improve the conductivity of pure silicon, arsenic (As) impurity atoms are diffused into the intrinsic Si lattice. Determine at what rate the arsenic (As) atoms would diffuse into the pure silicon (Si) at a temperature of 1100 °K.

Table T7. Diffusion constants D_0 and activation energies E for some crystals

Host crystal	Atom	D_o (cm^2/s)	E (eV)
Cu	Cu	0.20	2.04
Cu	Zn	0.34	1.98
Ag	Ag	0.40	1.91
Ag	Cu	1.2	2.00
Ag	Au	0.26	1.98
Ag	Pb	0.22	1.65
Na	Na	0.24	0.45
U	U	0.002	1.20

Host crystal	Atom	D_o (cm^2/s)	E (eV)
Si	Al	8.0	3.47
Si	Ga	3.6	3.51
Si	In	16.0	3.90
Si	As	0.32	3.56
Si	Sb	5.6	3.94
Si	Li	0.002	0.66
Si	Au	0.001	1.13
Ge	Ge	10.0	3.1

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USEFUL EQUATIONS AND CONSTANTS

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(1) $sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ $\quad cos\theta = \frac{1}{2}[\exp(i\theta)+\exp(-i\theta)]$ $\quad$ For $\theta << 1$: $cos\theta \cong 1-\frac{1}{2}\theta^2$

(2) $\boldsymbol{T} = u_1\boldsymbol{a}_1 + u_2\boldsymbol{a}_2 + u_3\boldsymbol{a}_3$

(3) $\boldsymbol{G} = v_1\boldsymbol{b}_1 + v_2\boldsymbol{b}_2 + v_3\boldsymbol{b}_3$

(4) $\boldsymbol{p} = \boldsymbol{r}\times\boldsymbol{t} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}(\boldsymbol{x}\ \boldsymbol{y}\ \boldsymbol{z}) = \begin{pmatrix} r_2t_3 - r_3t_2 \\ r_3t_1 - r_1t_3 \\ r_1t_2 - r_2t_1 \end{pmatrix}(\boldsymbol{x}\ \boldsymbol{y}\ \boldsymbol{z})$ where $\boldsymbol{r} = r_1\boldsymbol{x} + r_2\boldsymbol{y} + r_3\boldsymbol{z}$, $\boldsymbol{t} = t_1\boldsymbol{x} + t_2\boldsymbol{y} + t_3\boldsymbol{z}$

(5) $V_{min} = |\boldsymbol{a}_1 \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3)|$

(6) $\boldsymbol{b}_1 = 2\pi\frac{\boldsymbol{a}_2\times\boldsymbol{a}_3}{\boldsymbol{a}_1\cdot(\boldsymbol{a}_2\times\boldsymbol{a}_3)}$ $\quad \boldsymbol{b}_2 = 2\pi\frac{\boldsymbol{a}_3\times\boldsymbol{a}_1}{\boldsymbol{a}_1\cdot(\boldsymbol{a}_2\times\boldsymbol{a}_3)}$ $\quad \boldsymbol{b}_3 = 2\pi\frac{\boldsymbol{a}_1\times\boldsymbol{a}_2}{\boldsymbol{a}_1\cdot(\boldsymbol{a}_2\times\boldsymbol{a}_3)}$

(7) $2d\sin\theta = n\lambda$ $\quad \Delta\boldsymbol{k} = \boldsymbol{G}$ $\quad 2\boldsymbol{k}\cdot\boldsymbol{G} = \boldsymbol{G}^2$

(8) $U(R) = 4\epsilon\left[\left(\frac{\sigma}{R}\right)^{12} - \left(\frac{\sigma}{R}\right)^{6}\right]$ $\quad F(R) = -\,dU(R)/dR$

(9) $U_{tot} = -(2.15)(4N\varepsilon)$

(10) $F_s = C(u_{s+1}-u_s) - C(u_{s-1}-u_s)$

(11) $M\frac{d^2u_s}{dt^2} = C(u_{s+1}-u_s) - C(u_{s-1}-u_s)$

(12) $f(\in) = \frac{1}{exp\left[\frac{\in-\mu}{k_BT}\right]+1}$

(13) $\in_F = \frac{\hbar}{2m}\left(\frac{3\pi^2N}{V}\right)^{2/3}$

(14) $np = 4\left(\frac{k_BT}{2\pi\hbar^2}\right)^3 (m_em_h)^{3/2}\exp\left(\frac{-E_g}{k_BT}\right)$

(15) $n_i = p_i = 2\left(\frac{k_BT}{2\pi\hbar^2}\right)^{3/2} (m_em_h)^{3/4}\exp\left(\frac{-E_g}{2k_BT}\right)$

(16) $\mu = \frac{E_g}{2} + \frac{3}{4}k_BT\ \ln(m_h/m_e)$

(17) $\chi = -\frac{\mu_oNZe^2}{6m}\langle r^2\rangle$

(18) $\frac{n}{N-n} = exp\left(\frac{-E_V}{k_BT}\right)$

(19) $D = D_o exp\left(\frac{-E}{k_BT}\right)$

Quantity	Symbol	Value	CGS	SI
Velocity of light	c	2.997925	10^{10} cm s^{-1}	10^{8} m s^{-1}
Proton charge	e	1.60219	–	10^{-19} C
		4.80325	10^{-10} esu	–
Planck's constant	h	6.62620	10^{-27} erg s	10^{-34} J s
	$\hbar = h/2\pi$	1.05459	10^{-27} erg s	10^{-34} J s
Avogadro's number	N	6.02217×10^{23} mol^{-1}	–	–
Atomic mass unit	amu	1.66053	10^{-24} g	10^{-27} kg
Electron rest mass	m	9.10956	10^{-28} g	10^{-31} kg
Proton rest mass	M_p	1.67261	10^{-24} g	10^{-27} kg
Proton mass/electron mass	M_p/m	1836.1	–	–
Reciprocal fine structure constant $\hbar c/e^2$	$1/\alpha$	137.036	–	–
Electron radius e^2/mc^2	r_e	2.81794	10^{-13} cm	10^{-15} m
Electron Compton wavelength $\hbar/mc$	$\lambda\!\!\!^{-}_e$	3.86159	10^{-11} cm	10^{-13} m
Bohr radius $\hbar^2/me^2$	r_0	5.29177	10^{-9} cm	10^{-11} m
Bohr magneton $e\hbar/2mc$	μ_B	9.27410	10^{-21} erg G^{-1}	10^{-24} J T^{-1}
Rydberg constant $me^4/2\hbar^2$	R_∞ or Ry	2.17991	10^{-11} erg	10^{-18} J
		13.6058 eV		
1 electron volt	eV	1.60219	10^{-12} erg	10^{-19} J
	eV/h	2.41797×10^{14} Hz	–	–
	eV/hc	8.06546	10^{3} cm^{-1}	10^{5} m^{-1}
	eV/k_B	1.16048×10^{4} K	–	–
Boltzmann constant	k_B	1.38062	10^{-16} erg K^{-1}	10^{-23} J K^{-1}
Permittivity of free space	ϵ_0	–	1	$10^7/4\pi c^2$
Permeability of free space	μ_0	–	1	$4\pi \times 10^{-7}$ N/A^2