

## National Exams May 2014

### 07-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

#### NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Candidates may use a Casio or Sharp approved calculator. This is a closed book exam. No aids other than semi-log graph papers are permitted.
3. Any four questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.
4. All questions are of equal value.

**Question 1:**

A unity feedback system has an open-loop transfer function

$$G(s) = \frac{200K}{s(s+2)(s+5)}$$

Obtain a Bode plot for this system when  $K = 1$  and find the phase and gain margins.

**Question 2:**

Find the range of the parameter  $\alpha$  for which all roots of the given characteristic equations are in the left half-plane.

(a)  $s^3 + s^2 + s + \alpha = 0$

(b)  $s^3 + s^2 + \alpha s + 1 = 0$

(c)  $s^3 + \alpha s^2 + s + 1 = 0$

(d)  $\alpha s^3 + s^2 + s + 1 = 0$

**Question 3:**

Consider the system with the open-loop function

$$KG(s)H(s) = \frac{50K}{(s+1)(s+2)(s+10)}$$

(a) Sketch the root locus for this system for both positive and negative  $K$ .

(b) Locate all crossings of the  $j\omega$ -axis by the root locus, and find the value of  $K$  at each of these crossings.

(c) From the results in (b), state the complete range of  $K$  for which the system is stable.

**Question 4:**

Suppose that, in the system in Figure 1, the controller is a proportional type, that is,  $G_c(s) = K_p$ .

- (a) Find the transient-response terms for the case that  $K_p = 0.05$ .
- (b) Find the transient-response terms for the case that  $K_p = 0.5$ .
- (c) Find the minimum value of  $K$  for which the transient response will have the fastest decay.

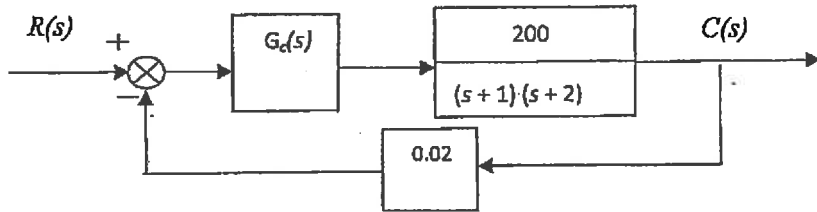


Figure 1

**Question 5:**

Consider the system shown in Figure 2. For each case given, find the steady-state error for (i) a unit step input; (ii) a unit ramp input. Assume in each case that the closed-loop system is stable.

(a)  $G(s) = \frac{10}{(s+1)(s+2)}$

(b)  $G(s) = \frac{10}{s(s+1)(s+5)}$

(c)  $G(s) = \frac{5(s+2)}{s^2(s+6)}$

(d)  $G(s) = \frac{6s^2 + 2s + 10}{s(s^2 + 3)}$



Figure 2

**Question 6:**

For the system of Figure 3, the input  $r(t) = 3 \cos 0.4t$  is applied at  $t = 0$ .

- (a) Find the steady-state system response.
- (b) Find the range of time  $t$  for which the system is in steady state.
- (c) Find the steady-state response for the input  $r(t) = 3 \cos 4t$ .

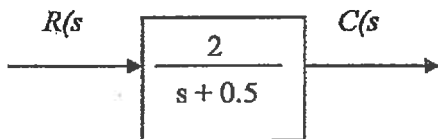


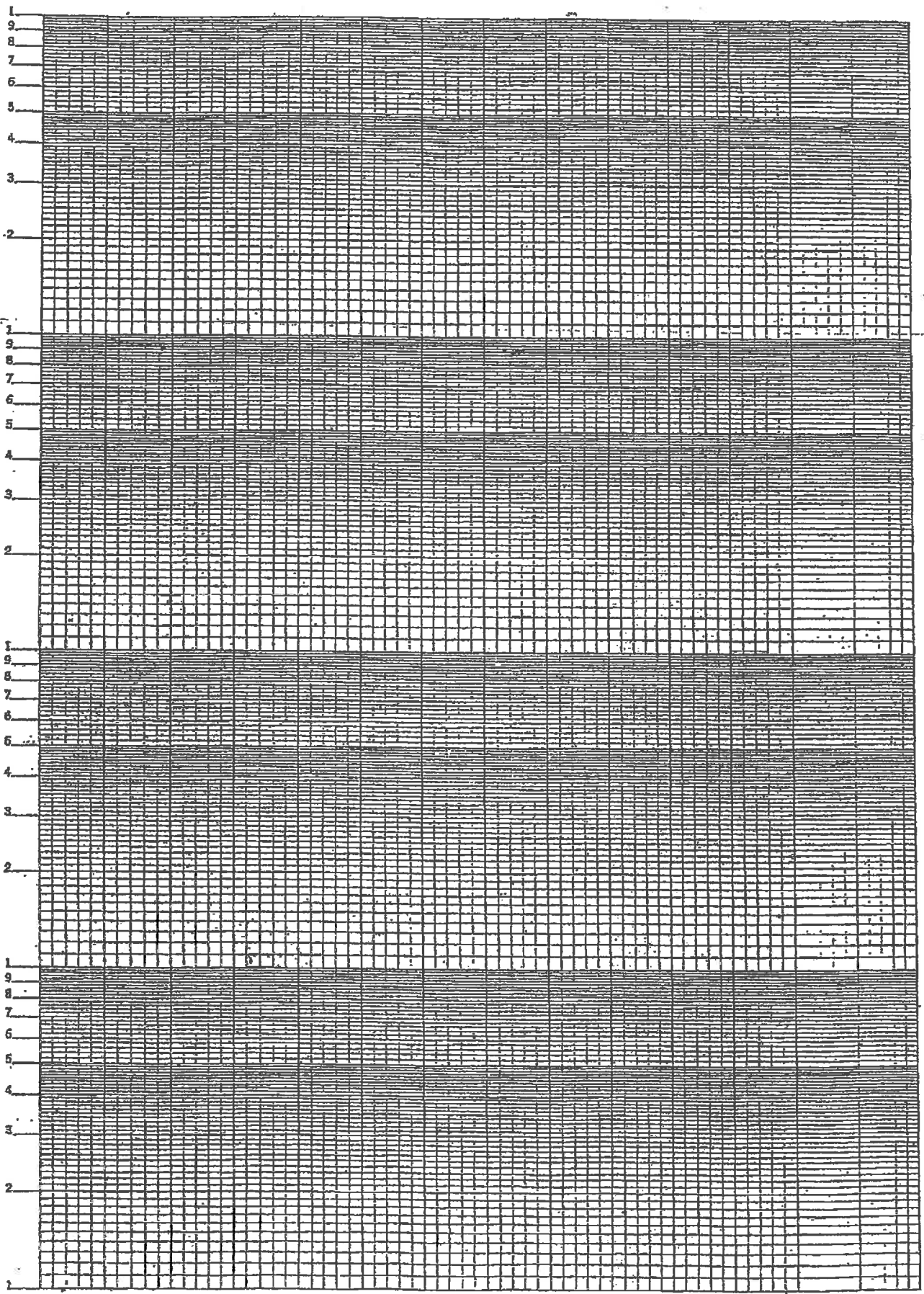
Figure 3

Laplace Transform Table

Laplace Transform $F(s)$	Time Function $f(t)$
1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$	Unit-step function $u_c(t)$
$\frac{1}{s^2}$	Unit-ramp function $t$
$\frac{n!}{s^{n+1}}$	$t^n$ ( $n =$ positive integer)
$\frac{1}{s + \alpha}$	$e^{-\alpha t}$
$\frac{1}{(s + \alpha)^2}$	$te^{-\alpha t}$
$\frac{n!}{(s + \alpha)^{n+1}}$	$t^n e^{-\alpha t}$ ( $n =$ positive integer)
$\frac{1}{(s + \alpha)(s + \beta)}$	$\frac{1}{\beta - \alpha}(e^{-\alpha t} - e^{-\beta t})$ ( $\alpha \neq \beta$ )
$\frac{s}{(s + \alpha)(s + \beta)}$	$\frac{1}{\beta - \alpha}(\beta e^{-\beta t} - \alpha e^{-\alpha t})$ ( $\alpha \neq \beta$ )
$\frac{1}{s(s + \alpha)}$	$\frac{1}{\alpha}(1 - e^{-\alpha t})$
$\frac{1}{s(s + \alpha)^2}$	$\frac{1}{\alpha^2}(1 - e^{-\alpha t} - \alpha t e^{-\alpha t})$
$\frac{1}{s^2(s + \alpha)}$	$\frac{1}{\alpha^2}(\alpha t - 1 + e^{-\alpha t})$
$\frac{1}{s^2(s + \alpha)^2}$	$\frac{1}{\alpha^2}\left[t - \frac{1}{\alpha} + \left(t + \frac{2}{\alpha}\right)e^{-\alpha t}\right]$

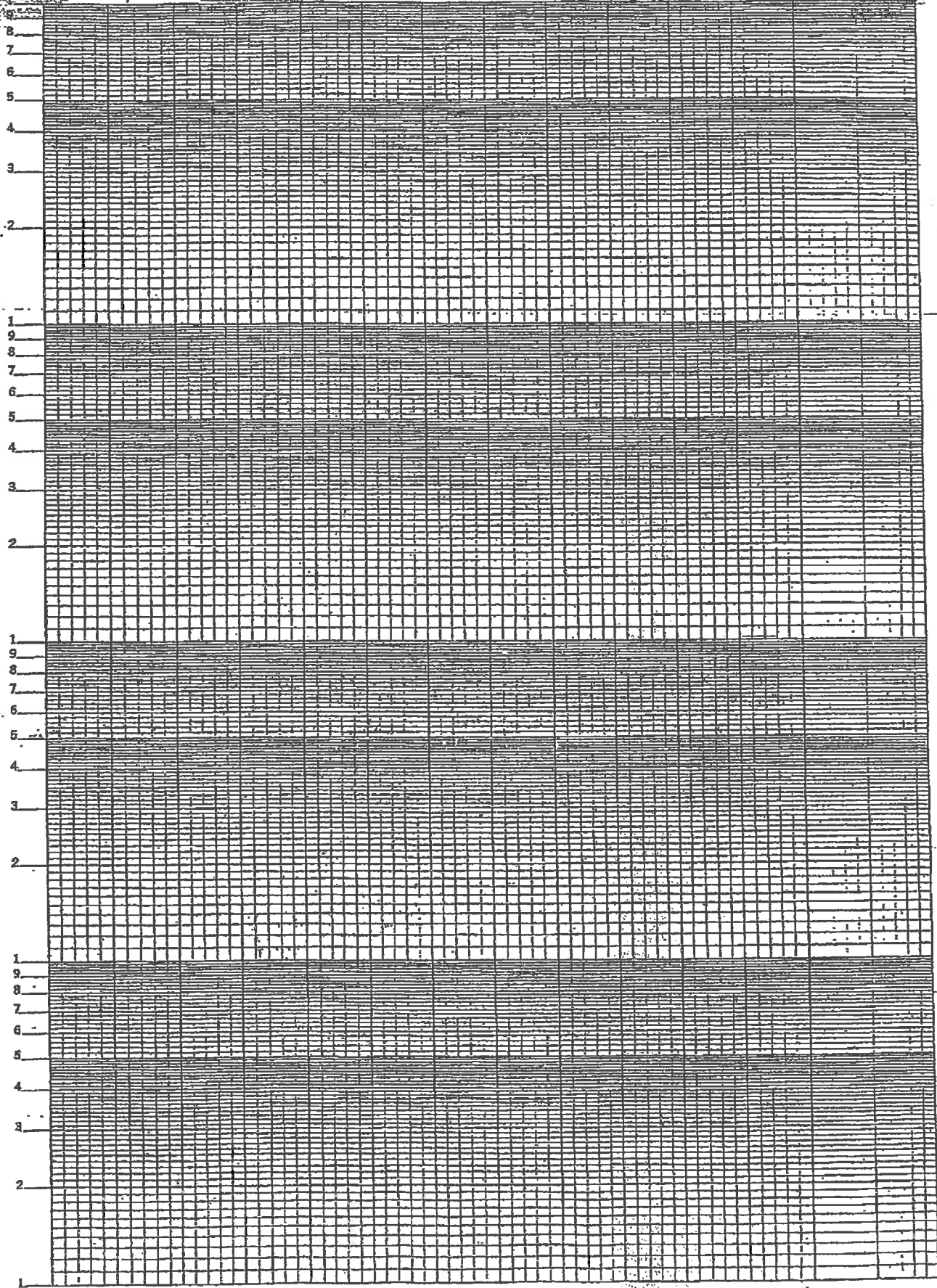
Laplace Transform Table (continued)

Laplace Transform $F(s)$	Time Function $f(t)$
$\frac{s}{(s + \alpha)^2}$	$(1 - \alpha t)e^{-\alpha t}$
$\frac{\omega_n^2}{s^2 + \omega_n^2}$	$\sin \omega_n t$
$\frac{s}{s^2 + \omega_n^2}$	$\cos \omega_n t$
$\frac{\omega_n^2}{s(s^2 + \omega_n^2)}$	$1 - \cos \omega_n t$
$\frac{\omega_n^2(s + \alpha)}{s^2 + \omega_n^2}$	$\omega_n \sqrt{\alpha^2 + \omega_n^2} \sin(\omega_n t + \theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n}{(s + \alpha)(s^2 + \omega_n^2)}$	$\frac{\omega_n}{\alpha^2 + \omega_n^2} e^{-\alpha t} + \frac{1}{\sqrt{\alpha^2 + \omega_n^2}} \sin(\omega_n t - \theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\frac{\omega_n}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1 - \zeta^2} t \quad (\zeta < 1)$
$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \cos^{-1} \zeta \quad (\zeta < 1)$
$\frac{s\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\frac{-\omega_n^2}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t - \theta)$ where $\theta = \cos^{-1} \zeta \quad (\zeta < 1)$
$\frac{\omega_n^2(s + \alpha)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\omega_n \sqrt{\frac{\alpha^2 - 2\alpha\zeta\omega_n + \omega_n^2}{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \tan^{-1} \frac{\omega_n \sqrt{1 - \zeta^2}}{\alpha - \zeta\omega_n} \quad (\zeta < 1)$
$\frac{\omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$1 - \frac{2\zeta}{\omega_n} + \frac{1}{\omega_n^2 \sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \cos^{-1}(2\zeta^2 - 1) \quad (\zeta < 1)$



Semi-Logarithmic  
4 Cycles x 10 to the inch

12-10-47



Semi-Logarithmic  
4 Cycles x 14 to the Inch