NATIONAL EXAMINATIONS MAY 2014

04-BS-2

PROBABILITY AND STATISTICS

2 HOURS DURATION

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumption made.

2. "Closed Book" – no-aids other than

   (i) A Casio or Sharp approved calculator
   (ii) ONE hand-written information sheet (8.5"x11"), filled on both sides.

3. Any 5 questions constitute a complete paper. Only 5 questions will be marked.

4. All questions are of equal value.

5. Statistical tables of the normal, t, chi-square and F distributions are provided.

6. Questions involving hypothesis testing must be clearly formulated.

Marking Scheme

1. (a) 5 marks (b) 5 marks (c) 5 marks (d) 5 marks
2. (A) (a) 5 marks (b) 5 marks ; (B) (a) 5 marks ;(b) 5 marks
3. (A) (a) 5 marks (b) 5 marks ; (B) (a) 5 marks (b) 5 marks
4. (a) 5 marks (b) 5 marks (c) 5 marks (d) 5 marks
5. (a) 7 marks (b) (i) 7 marks (ii) 6 marks
6. (A) (a) 5 marks (b) 5 marks ; (B) (a) 5 marks (b) 5 marks
7. (a) 10 marks (b) 10 marks
8. (a) 5 marks (b) 5 marks (c) 5 marks (d) 5 marks

1. The volume X of fabric softener contained in the plastic bottles sold by a supermarket chain is a normally distributed random variable with a mean of 4,850mL and a standard deviation of 40mL.

(a) Find the probability that a randomly selected bottle will contain less than 4,900mL of fabric softener. Write down the probability density function of X. Then draw the probability density function of X, neatly and clearly, and indicate the area that corresponds to this probability.

(b) Compute the probability that the volume X of fabric softener contained in a randomly selected bottle differs from the mean by more than 60mL. Then draw, clearly and neatly, the probability density of X and indicate the area that corresponds to this probability.

(c) Let M represent the mean volume of fabric softener contained in a random sample of four bottles. (i) Find the mean and variance of the probability distribution of M. (ii) Write down the probability density function of M. (iii) Draw, neatly and clearly, the probability density function of X and M on the same diagram. (iv) Compute the probability that M is more than 4,840mL.

(d) Let T be the total volume of fabric softener contained in a random sample of nine bottles. Find E(T) and Var(T). Then compute the probability that T exceeds 43,680mL. Draw the probability density function of T and indicate the area that corresponds to this probability.

2. (A) Information gathered by the Central Weather Office over a long period reveals that the number of severe snowstorms experienced by a large urban centre follows the Poisson law with an average of three such storms per year.

(a) What is the probability that the number of severe snowstorms next year will be smaller than four?

(b) Compute the probability that in a two-year period the urban centre will experience more than four but fewer than eight severe snowstorms.

2. (B) The digital cameras manufactured by Marvelcam come in either a 10 megapixel version or a 20 megapixel version. The owner of a camera store received a shipment of fifteen cameras from Marvelcam: six 10 megapixel and nine 20 megapixel cameras. An employee randomly selects five of these cameras and stores them behind the counter; he then places the remaining ten cameras in the storeroom. Let X denote the number of 10 megapixel cameras stored behind the counter.

(a) Compute the probability that X is larger than two.

(b) Write down the probability distribution of X.

3.(A) An investigation carried out by the quality control department of Excellent Laboratory Supplies revealed that 25% of the safety glasses manufactured by the company do not meet the rigid standards demanded by the senior management.

(a) Compute the probability that in a random sample of 10 safety glasses produced on a given month fewer than three fail to meet the standards demanded.

(b) Use an appropriate approximation to compute the probability that in a random sample of 1,600 safety glasses more than 1,230 will meet the rigid standards demanded.

3.(B) A committee composed of the four most senior engineers of Antarctica Hydro – Mr. Allyson, , Mrs. Caballero, Mr. Ethnarch and Mrs. Gamelle – was formed to make an important decision on the construction site of a new power station. Assume that each member reaches a decision independently of the others. Assume also that the probability that Mr. Allyson, Mrs. Caballero, Mr. Ethnarch and Mrs. Gamelle are in favour of the construction site under consideration is 0.40, 0.80, 0.6 and 0.9 respectively. If X denotes the number of committee members who are in favour of the construction site under consideration find

(a) the probability distribution of X and

(b) the mean and variance of the probability distribution of X.

4. The warehouse of an online retailer has four employees. Employee A takes care of 30% of the orders, while employees B, C and D take care of 25%, 20% and 25% of the orders respectively. Data also show that the probability that employee A makes a mistake while filling an order is 1%, while the corresponding probabilities for employees B, C and D are 1.2%, 1.4% and 1.1% respectively.

(a) Let M denote the event "Employee does not make a mistake while filling an order". Also let M^C denote the complement of M. Draw a neat and properly labelled tree diagram indicating all the relevant probabilities using the symbols A, B, C, D, M and M^C.

(b) Compute the following probabilities:

(i) $Pr(M)$; (ii) $Pr(B \cap M)$; (iii) $Pr(D \cap M^C)$

(c) The supervisor of the warehouse receives a complaint from a customer who received a wrong item. What is the probability that employee C is the one who made the mistake?

(d) Assume that the employees of the warehouse filled fifteen orders in a given hour. What is the probability that at least fourteen orders were filled correctly?

5. It is believed that a minor technical modification to the carburetor of the vans manufactured by Toro Motors will lead to an improvement in the fuel consumption. Tests carried out on twelve of these vans yielded the following results in kms/litre:

$$\sum X = 157.20 \qquad \sum X^2 = 2{,}062.07$$

(a) Find the 99% confidence limits of (i) the true mean and (ii) the true standard deviation of the probability distribution of X. Assume that X is a normally distributed random variable.

(b) The extensive data accumulated by the design department of Toro Motors indicate that the mean and standard deviation of the fuel consumption without the modification mentioned above are 12.2kms/litre and 0.9kms/litre respectively.
(i) Test the hypothesis that the mean fuel consumption using the new technical modification is not significantly different from 12.2kms/litre. Let $\alpha = 0.05$.
(ii) Test the hypothesis that the true standard deviation of the fuel consumption is not significantly different from 0.9kms/litre. Let $\alpha = 0.05$.

6. (A) A random sample of 1,800 measurements of the weight W of the hand luggage carried by the passengers of Stratosphere Airlines yielded a mean weight of 5.850kgs and a standard deviation of 1.350kgs.

(a) Test the hypothesis that the mean weight of the hand luggage under study is larger than 5.5kgs. Let $\alpha = 0.05$. Assume that W is a normally distributed random variable.

(b) The following is an interesting and useful way of finding an approximate $(1-\alpha)100\%$ confidence interval of the standard deviation σ when the sample is large:

$$\frac{s}{1+\frac{z_{\alpha/2}}{\sqrt{2n}}} < \sigma < \frac{s}{1-\frac{z_{\alpha/2}}{\sqrt{2n}}}$$

Use this result to test the hypothesis at the $\alpha = 0.05$ level that the true standard deviation σ is not significantly different from 1.2kgs.

6. (B) A nationwide survey carried out on behalf of the City Dwellers Association revealed that 1,960 households out of a random sample of 2,800 were dissatisfied with the level of municipal services provided by the city.

(a) Test the hypothesis that the proportion of households who are not satisfied with the level of municipal services is not significantly different from 0.65. Let $\alpha = 0.05$.

(b) How large should the sample be if we wish to know the true proportion of dissatisfied households with an error of 0.01 and 99% confidence?

7. Professor Sparks, a respected professor of Electrical Engineering, was hired to test the breakdown voltage of two equally priced insulating materials obtained from two manufacturers. Tests were carried out on a random sample of twelve samples obtained from each manufacturer. The tests yielded the following results: (Note: The results are in kV/mm)

	Manufacturer A	Manufacturer B
Sample size	$n_A = 12$	$n_B = 12$
Sample Mean	$m_A = 26.25$	$m_B = 25.95$
Sample Standard deviation	$s_A = 0.6$	$s_B = 0.7$

(a) Test the hypothesis that the variability of the breakdown voltage of the insulating material obtained from manufacturer A is not significantly different from that obtained from manufacturer B. Let $\alpha=0.05$. State any assumptions you need to make.

(b) Test the hypothesis that the mean breakdown voltage of the insulating material obtained from manufacturer A is not significantly different from that obtained from manufacturer B. Let $\alpha=0.05$.

8. The following information was obtained from a study carried out by professor Socrates. Y represents the mark, out of hundred, while X represents the number of hours that a candidate devoted to the study of a certain professional examination. The results were obtained from a random sample of twenty-one candidates.

$$\sum_{i=1}^{n} X_i = 2{,}940.0 \quad ; \quad \sum_{i=1}^{n} X_i^2 = 436{,}100.0 \quad ; \quad \sum_{i=1}^{n} Y_i = 1{,}470.0;$$

$$\sum_{i=1}^{n} Y_i^2 = 110{,}900.0 \quad ; \quad \sum_{i=1}^{n} X_i Y_i = 219{,}100.0 \quad ; \quad n = 21$$

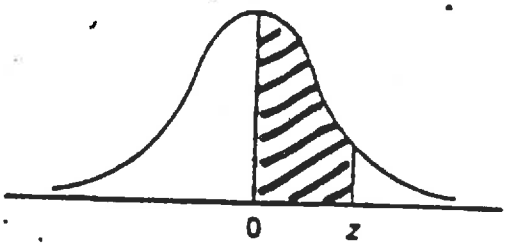
(a) Compute Cov(X,Y) and the coefficient of correlation r of X and Y.

(b) Test the null hypothesis that the true coefficient of correlation ρ is not significantly different from 0.9. Let $\alpha=0.05$.

(c) It is believed that Y and X are related by an equation of the form $Y=\beta_0 + \beta_1 X + \varepsilon$. Write down the normal equations of the least squares line and then compute the estimates b_0 and b_1 of β_0 and β_1 respectively.

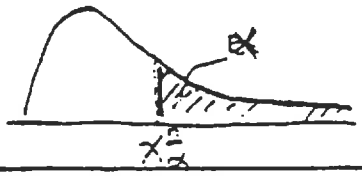
(d) Compute the error sum of squares and use this information to find the 95%· confidence limits of β_1.

NORMAL DISTRIBUTION TABLE

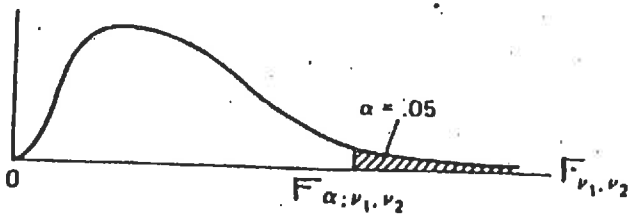


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

THE CHI-SQUARE DISTRIBUTION



df	Probability that chi-square value will be exceeded							
	.995	.990	.975	.950	.050	.025	.010	.005
1	---	---	---	.004	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	21.03	23.34	26.22	23.30
13	3.57	4.11	5.01	5.89	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	41.34	44.46	48.23	50.99
29	13.12	14.26	16.05	17.71	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	79.08	83.30	88.33	91.95
70	43.28	45.44	48.76	51.74	90.53	95.02	100.43	104.22
80	51.17	53.54	57.15	60.39	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	124.34	129.56	135.81	140.17



Upper 5% points

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.2	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00