

National Exams December 2014

07-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Candidates may use a Casio or Sharp approved calculator. This is a closed book exam. No aids other than semi-log graph papers are permitted.
3. Any four questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Question 1:

Accurately sketch the root locus for the system shown in Figure 1, and give the range of $0 < K < \infty$ for which the system is stable.

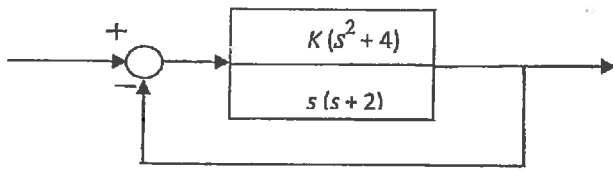


Figure 1

Question 2:

Sketch the Bode diagrams for the given transfer functions.

$$(a) G(s) = \frac{1}{s^2(s+1)}$$

$$(b) G(s) = \frac{s}{(s+1)(s+1)}$$

Question 3:

For the system of Figure 2, the input $r(t) = 3 \cos 0.4t$ is applied at $t = 0$.

- (a) Find the steady-state system response.
- (b) Find the range of time t for which the system is in steady state.
- (c) Find the steady-state response for the input $r(t) = 3 \cos 4t$.

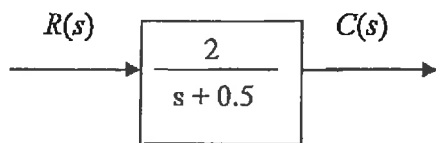


Figure 2

Question 4:

Use the Routh-Hurwitz criterion to determine the number of roots in the left half-plane, the right-half plane, and on the imaginary axis for the given characteristic equations.

(a) $s^4 + 2s^2 + 1 = 0$

(b) $s^4 + s^3 + 5s^2 + 5s + 2 = 0$

(c) $s^4 + 2s^3 + 3s^2 + 2s + 5 = 0$

Question 5:

Consider the system of Figure 3. Assume that all initial conditions are zero.

- If the disturbance $D(s)$ is zero and $R(s)$ is not zero, find the transient-response term in the system output as a function of the gain K .
- If the disturbance $D(s)$ is not zero and $R(s)$ is zero, find the transient-response term in the system output as a function of the gain K .
- Find the value of K that will give the closed-loop system a time constant of 1 s.

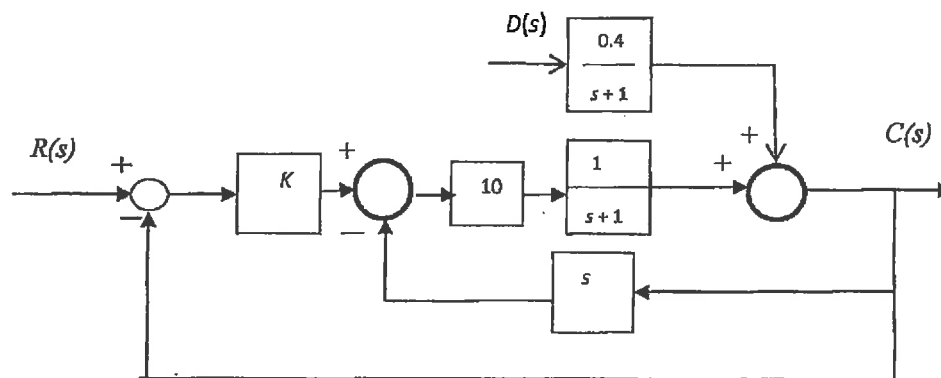


Figure 3

Question 6:

In the motor position servo of Fig. 4, let $G(s) = 1/[s(s + 1)]$ represent the motor and load and $G_c = K$ the controller/amplifier. Calculate the steady-state errors of the system for unit step and unit ramp signals applied in turn to reference input R .

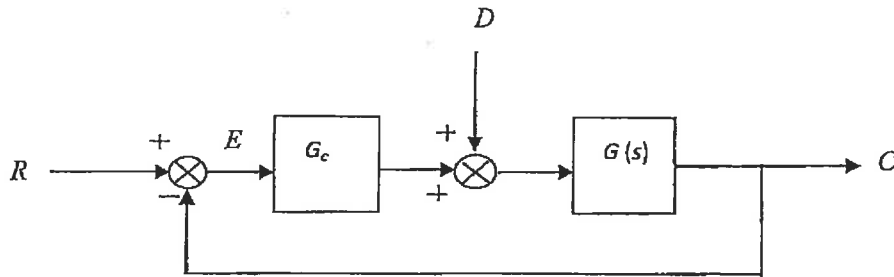


Figure 4

Laplace Transform Table

Laplace Transform $f(s)$	Time Function $f(t)$
1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$	Unit-step function $u(t)$
$\frac{1}{s^2}$	Unit-ramp function t
$\frac{n!}{s^{n+1}}$	t^n ($n = \text{positive integer}$)
$\frac{1}{s + \alpha}$	$e^{-\alpha t}$
$\frac{1}{(s + \alpha)^2}$	$te^{-\alpha t}$
$\frac{n!}{(s + \alpha)^{n+1}}$	$t^n e^{-\alpha t}$ ($n = \text{positive integer}$)
$\frac{1}{(s + \alpha)(s + \beta)}$	$\frac{1}{\beta - \alpha}(e^{-\alpha t} - e^{-\beta t})$ ($\alpha \neq \beta$)
$\frac{s}{(s + \alpha)(s + \beta)}$	$\frac{1}{\beta - \alpha}(\beta e^{-\beta t} - \alpha e^{-\alpha t})$ ($\alpha \neq \beta$)
$\frac{1}{s(s + \alpha)}$	$\frac{1}{\alpha}(1 - e^{-\alpha t})$
$\frac{1}{s(s + \alpha)^2}$	$\frac{1}{\alpha^2}(1 - e^{-\alpha t} - \alpha t e^{-\alpha t})$
$\frac{1}{s^2(s + \alpha)}$	$\frac{1}{\alpha^2}(\alpha t - 1 + e^{-\alpha t})$
$\frac{1}{s^2(s + \alpha)^2}$	$\frac{1}{\alpha^2}\left[t - \frac{1}{\alpha} + \left(t + \frac{2}{\alpha}\right)e^{-\alpha t}\right]$

Laplace Transform Table (continued)

Laplace transform (s)	time function (t)
$\frac{s}{(s + \alpha)^2}$	$(1 - \alpha t)e^{-\alpha t}$
$\frac{\omega_n^2}{s^2 + \omega_n^2}$	$\sin \omega_n t$
$\frac{s}{s^2 + \omega_n^2}$	$\cos \omega_n t$
$\frac{\omega_n^2}{s(s^2 + \omega_n^2)}$	$1 - \cos \omega_n t$
$\frac{\omega_n^2(s + \alpha)}{s^2 + \omega_n^2}$	$\omega_n \sqrt{\alpha^2 + \omega_n^2} \sin(\omega_n t + \theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n}{(s + \alpha)(s^2 + \omega_n^2)}$	$\frac{\omega_n}{\alpha^2 + \omega_n^2} e^{-\alpha t} + \frac{1}{\sqrt{\alpha^2 + \omega_n^2}} \sin(\omega_n t - \theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\frac{\omega_n}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1 - \zeta^2} t \quad (\zeta < 1)$
$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \cos^{-1} \zeta \quad (\zeta < 1)$
$\frac{s\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\frac{-\omega_n^2}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t - \theta)$ where $\theta = \cos^{-1} \zeta \quad (\zeta < 1)$
$\frac{\omega_n^2(s + \alpha)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\omega_n \sqrt{\frac{\alpha^2 - 2\alpha\zeta\omega_n + \omega_n^2}{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \tan^{-1} \frac{\omega_n \sqrt{1 - \zeta^2}}{\alpha - \zeta\omega_n} \quad (\zeta < 1)$
$\frac{\omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$1 - \frac{2\zeta}{\omega_n} + \frac{1}{\omega_n^2 \sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \cos^{-1}(2\zeta^2 - 1) \quad (\zeta < 1)$

46 6010

 SEMI-LOGARITHMIC 4 CYCLES X 7/8 DIVISIONS
KEUFFEL & ESSER CO. MADE IN U.S.A.

