Professional Engineers of Ontario

Annual Examinations – **December 2014**

07-Elec-B3
Digital Communication Systems

3 Hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.
2. This is a closed book exam. A PEO-approved non-programmable calculator is permitted.
3. There are **5 questions** on this exam. **Any 4 questions constitute a complete paper.** Only the first 4 questions as they appear in your answer book will be marked.
4. Marks allocated to each question are noted in the left margin. A complete paper is worth 100 marks.

(25 marks) **Question 1.** This question concerns link budgeting.

(10 marks) a. Consider a wireless system with transmitter power of 500 mW, antenna gains of 9 dB, receiver losses of 6 dB, receiver noise figure of -168 dBm/Hz, a bandwidth of 10 MHz, and a fading margin requirement of 6 dB. Aside from free-space losses, no other gains or losses are present other than path loss. If the receiver requires a signal-to-noise ratio of at least 2 dB, what is the maximum allowed path loss (in dB)?

(10 marks) b. Using a free-space path loss of $30 \log_{10}(4 \pi df/c)$, where d represents the distance from transmitter to receiver, f represents the carrier frequency, and c represents the speed of light ($c = 3.0 \times 10^8$ m/s), and assuming a carrier frequency of 1.5 GHz, is the signal-to-noise criterion satisfied when d = 200 m? Show all work.

(5 marks) c. If a radio transmits at a power level of 20 dBm, what is the radio's power output in W?

(25 marks) **Question 2.** This question concerns modulation schemes.

(7 marks) a. Briefly explain how a signal is transmitted using the rectangular 16QAM constellation.

(7 marks) b. Suppose the available system bandwidth is 1 MHz. Using 16QAM symbols, and pulses that satisfy the Nyquist criterion, what is the maximum data rate (in bits per second) at which information can be transmitted over this link?

(7 marks) c. The information-theoretic capacity of a system (in bits per second) is given by $C = B \log_2(1 + S)$, where B is the system bandwidth (in Hz), and S is the signal-to-noise ratio. For what value of S can the data rate from part b be achieved? (Assume a value of 4 Mbit/s if you did not complete part b.)

(4 marks) d. Suppose a vendor approaches you and tells you about a new technology which, according to the vendor's claim, allows data transmission at a rate faster than the capacity C. Are the vendor's claims reasonable? Why or why not?

(25 marks) **Question 3.** This question concerns source coding.

(10 marks) a. You are given a source with six letters: A, B, C, D, E, F. The probabilities of these letters are: Pr(A) = 0.11; Pr(B) = 0.05; Pr(C) = 0.28; Pr(D) = 0.17; Pr(E) = 0.25; Pr(F) = 0.14. Find a Huffman code for this source.

(5 marks) b. What is the entropy of the source in part a?

(5 marks) c. What is a "prefix code"? Is a Huffman code a prefix code? Explain.

(5 marks) d. Give one advantage and one disadvantage of Huffman codes.

(25 marks) **Question 4.** This question concerns signal detection.

(5 marks) a. Consider signals $s_0(t)$ and $s_1(t)$, which are used to modulate the binary symbols "0" and "1", respectively, where

$$s_0(t) = \begin{cases} 1, & 0 \leq t \leq T; \\ 0 & \text{elsewhere} \end{cases}$$

and $s_1(t) = - s_0(t)$. Sketch the two signals, and sketch the impulse response of the matched filter $m(t)$, assuming the filter is matched to $s_0(t)$, and assuming the filter output is sampled at time T.

(5 marks) b. Sketch the convolution of $s_0(t)$ and $m(t)$ as a function of t.

(5 marks) c. At the sampling instant (time T), the matched filter output is corrupted by additive Gaussian noise with zero mean and variance σ^2. Give the optimal decision rule assuming that 0 and 1 are equiprobable.

(10 marks) d. Given that

$$\frac{1}{2}\text{erfc}\left(\frac{t-\mu}{\sqrt{2\sigma^2}}\right) = \int_t^\infty \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx$$

and given your decision rule from part c, express the probability of error *given than a 1 was sent* in terms of erfc.

(25 marks) **Question 5.** This question concerns error-control coding.

(5 marks) a. Briefly discuss the difference between a block code and a convolutional code.

(5 marks) b. What is meant by the "minimum Hamming distance" of a code? How is the minimum Hamming distance related to the number of errors a code can correct?

(5 marks) c. Consider a binary Hamming code with the following parity check matrix:

Give the corresponding generator matrix.

(10 marks) d. Give an example illustrating how to correct a single bit error using the Hamming code from part c.