NATIONAL EXAMINATIONS DECEMBER 2014

04-BS-5 ADVANCED MATHEMATICS

3 Hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumption made.

2. Candidates may use one of the approved Casio or Sharp calculators. This is a Closed Book Exam. However, candidates are permitted to bring **ONE** aid sheet (8.5"x11") written on both sides.

3. Any five (5) questions constitute a complete paper. Only the first five answers as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme

1. 20 marks

2. (A) 14 marks ; (B) 6 marks

3. (a) 5 marks ; (b) 9 marks ; (c) 3 marks ; (d) 3 marks

4. (A) 14 marks ; (B) 6 marks

5. 20 marks

6. (a) 10 marks ; (b) 10 marks

7. (a) 10 marks ; (b) 10 marks

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1.Find two linearly independent solutions about the ordinary point x=0 for the following differential equation

$$(x^2 + 1)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 2y = 0$$

2.(A) Find the Fourier series expansion of the periodic function f(x) of period p=2.

$$f(x) = \begin{array}{ll} x(1+x) & -1 < x < 0 \\ \\ x(1-x) & 0 < x < 1 \end{array}$$

2.(B) Use the result obtained in (A) to prove the following

$$\frac{\pi^3}{32} = \sum_{n=1}^{\infty} \frac{(-1)^{(n-1)}}{(2n-1)^3}$$

3.Consider the following function where a is a positive constant

$$f(x) = \begin{array}{ll} \frac{1}{2a}\exp(x/a) & x < 0 \\ \\ \frac{1}{2a}\exp(-x/a) & x > 0 \end{array}$$

   (a) Compute the area bounded by f(x) and the x-axis. Graph f(x) against x for $a = 2$ and $a = 0.5$.
   (b) Find the Fourier transform F(ω) of f(x).
   (c) Graph F(ω) against ω for the same two values of a mentioned in (a).
   (d) Explain what happens to f(x) and F(ω) when a tends to zero.

   Note: $F(\omega) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{\infty} f(x)\exp(-i\omega x)dx$

4.(A)Set up Newton's divided difference formula for the data tabulated below and derive the polynomial of highest possible degree.

x	−5	−4	−3	−1	0	2	3
f(x)	−175	−156	−65	93	100	0	−23

4(B).Use the forward difference formulas supplied below to find the approximate value of the first, second, third and fourth derivative of the function f(x) tabulated below at x = −2. (Hint: Let $x_0 = -2$ and h = 1)

x	−4	−3	−2	−1	0	1	2	3	4	5
f(x)	−55	−7	11	11	5	5	23	71	161	305

$$f'(x_0) \approx \frac{1}{12h}\left[-25f(x_0)+48f(x_0+h)-36f(x_0+2h)+16f(x_0+3h)-3f(x_0+4h)\right]$$

$$f''(x_0) \approx \frac{1}{12h^2}\left[35f(x_0)-104f(x_0+h)+114f(x_0+2h)-56f(x_0+3h)+11f(x_0+4h)\right]$$

$$f'''(x_0) \approx \frac{1}{2h^3}\left[-5f(x_0)+18f(x_0+h)-24f(x_0+2h)+14f(x_0+3h)-3f(x_0+4h)\right]$$

$$f''''(x_0) \approx \frac{1}{h^4}\left[f(x_0)-4f(x_0+h)+6f(x_0+2h)-4f(x_0+3h)+f(x_0+4h)\right]$$

5. The following results were obtained in a certain experiment:

x	1	1.50	2	2.50	3	3.50	4	4.50	5
y	230	345	436	527	500	773	864	955	1,070

Use Romberg's algorithm to obtain an approximation of the area bounded by the unknown curve represented by the table and the lines x = 1, x = 5 and the x-axis.

Note: The Romberg algorithm produces a triangular array of numbers, all of which are numerical estimates of the definite integral $\int_a^b f(x)dx$. The array is denoted by the following notation:

R(1,1)
R(2,1)   R(2,2)
R(3,1)   R(3,2)   R(3,3)
R(4,1)   R(4,2)   R(4,3)   R(4,4)

where

$$R(1,1) = \frac{H_1}{2}\left[f(a)+f(b)\right] \ ;$$

$$R(k,1) = \frac{1}{2}\left[R(k-1,1)+H_{k-1}\sum_{n=1}^{n=2^{k-2}}f(a+(2n-1)H_k)\right] \ ; \quad H_k = \frac{b-a}{2^{k-1}}$$

$$R(k,j) = R(k,j-1)+\frac{R(k,j-1)-R(k-1,j-1)}{4^{j-1}-1}$$

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6.(A) The equation $5^x + 6x - 3 = 0$ has a root close to $x_0 = 1.5$. Use Newton's method three times to find a better approximation of this root. (Note: Carry seven digits in your calculations)

6.(B) Consider the equation $x^3 - 7x^2 + 11x - 3 = 0$. This equation can be written in the form $x = g(x)$ in several ways. Find the root that is close to $x_0 = 1.7$ using the form $x = (x^3 + 11x - 3)/7x$ five times. Explain why this form converges to the root. (Note: Carry seven digits in your calculations).

7. The matrix $A = \begin{bmatrix} 2 & -4 & 6 \\ 4 & -11 & 0 \\ -1 & 4 & 10 \end{bmatrix}$ can be written as the product of a lower

triangular matrix $L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$ and an upper triangular matrix

$U = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$,   that is A=LU.

(a) Find L and U.

(b) Use the results obtained in (a) to solve the following system of three linear equations:

$$\begin{aligned} 2x &- 4y + 6z &= 7 \\ 4x &- 11y &= 23 \\ -x &+ 4y + 10z &= -12 \end{aligned}$$