

National Exams
04-CHEM-B1, Transport Phenomena
3 hours duration

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. The examination is an OPEN BOOK EXAM.
3. Candidates may use any **non-communicating** calculator.
4. **Not all** problems are of equal weight.
5. **Answer all five questions.**
6. State all assumptions clearly.
7. The various conservation equations (continuity, momentum and shear stress-velocity gradient relationships, energy, and species conservation) are given in Tables 1-5 appended to this paper. These equations are also available in Brodkey, R.S. and Hershey H.C. (1988) *Transport Phenomena – A Unified Approach*.

04-CHEM-B1, Transport Phenomena

- Q1. [15 marks] Making use of the continuity equation for an incompressible fluid flowing at steady state, show that a flow defined by the velocity field:

$$\vec{u} = (4t + 4x + 4y)\vec{i} + (t - 2y - 2z)\vec{j} + (t + x - 2z)\vec{k}$$

is possible in which \vec{i} , \vec{j} and \vec{k} are the unit vectors in the x , y , and z directions.

- Q2. [30 marks overall] Consider the steady, low speed flow of a viscous isothermal fluid of density ρ and viscosity μ , between two infinitely long, parallel, vertical plates, spaced a distance h apart. The plate on the left (at $x = 0$) is stationary, whereas the plate on the right (at $x = h$) moves upward as constant speed, u_0 .

- (a) [15 marks] With reference to Fig. 1, and starting with the appropriate form of the Navier-Stokes equation, show that the velocity distribution is given by:

$$u_z = \frac{\rho g h^2}{2\mu} \left[\left(\frac{x}{h} \right)^2 - \left(\frac{x}{h} \right) \right] + u_0 \left(\frac{x}{h} \right)$$

- (b) [5 marks] Derive an expression for the velocity in the middle of the channel.
 (c) [10 marks] Show that for the net mass flow rate to be zero, the right hand plate must move at a velocity given by:

$$u_0 = \frac{\rho g h^2}{6\mu}$$

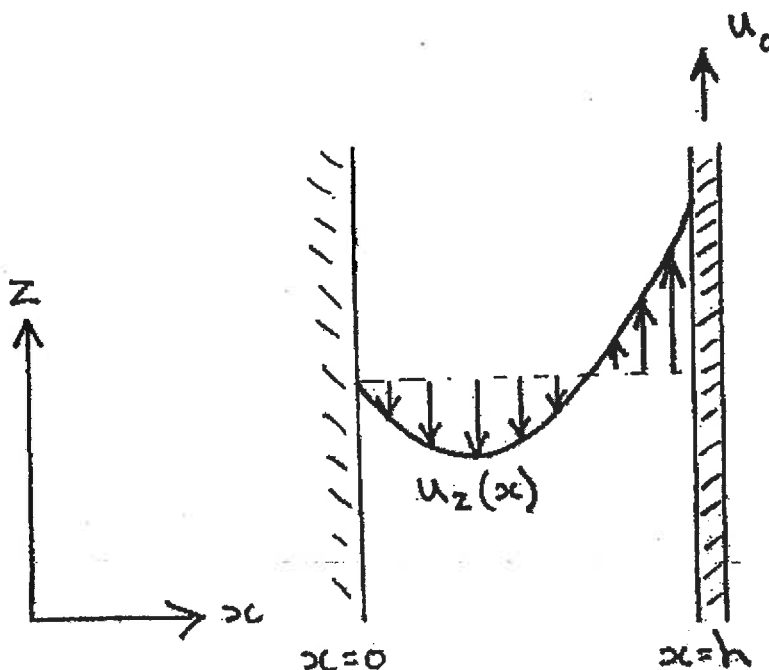


Fig. 1: Fully developed flow between vertical plates with one plate moving

04-CHEM-B1, Transport Phenomena

Q3. [25 marks overall] Consider a hollow sphere of inner radius R_1 and outer radius R_2 .

(i) [15 marks] If the inner surface is at the higher temperature T_1 and the outer surface is at the lower temperature T_2 , starting with the appropriate form of the energy equation show that the temperature distribution throughout the shell is given by:

$$T = T_1 - \left\{ \frac{(T_1 - T_2) \cdot R_2 \cdot (r - R_1)}{r \cdot (R_2 - R_1)} \right\}$$

(ii) [10 marks] Furthermore, show that the heat loss at the outer surface of the sphere is given by:

$$\dot{Q} = -k \cdot 4\pi R_2^2 \cdot \frac{R_1 R_2 (T_1 - T_2)}{R_2^2 (R_1 - R_2)}$$

Q4. [15 marks overall] The instantaneous reaction $A \rightarrow 2B$ occurs on the outer surface of a long cylinder of length L with a diameter of 0.03 m. The ambient environment contains approximately 100 mol% A and is at 300 K and 1 atm.

(i) [10 marks] Show that an expression for the mole fraction profile is:

$$y_A = (1 + y_{As}) \left(\frac{r}{R} \right)^{\left(\frac{\dot{N}_A}{2\pi L c D_{AB}} \right)} - 1$$

in which y_{As} is the mole fraction of A at the surface of the cylinder and \dot{N}_A is the molar flow rate of A.

(ii) [5 marks] If $D_{AB} = 1.6 \times 10^{-5} \text{ m}^2/\text{s}$ and the boundary layer thickness is 1 cm, determine the rate of formation of B per unit length of cylinder.

Q5. [15 marks] Oxygen is transferred from the lung cavity, across the lung tissue, to the network of blood vessels on the opposite side. The lung tissue may be viewed as a plane of thickness L . The inhalation process maintains a constant molar oxygen concentration, C_{Ai} , on the inner surface of the tissue. Due to absorption of oxygen by the blood, a constant concentration of C_{Ao} is maintained on the other surface. Oxygen is consumed through metabolic processes in the lung tissue at a constant volumetric rate of $-k_0$. Show that an expression for the oxygen concentration profile in the lung tissue is:

$$C_A = C_{Ai} + \frac{k_0}{2D_{AB}} (z^2 - zL) + \frac{(C_{Ao} - C_{Ai})}{L} z$$

04-CHEM-B1, Transport Phenomena

APPENDIX

Useful equations

Table 1: The Continuity Equation

$[\partial\rho/\partial t + (\nabla \cdot \rho\vec{u})] = 0$		(1)
Rectangular coordinates (x, y, z)		
$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u_x) + \frac{\partial}{\partial y}(\rho u_y) + \frac{\partial}{\partial z}(\rho u_z) = 0$		(1a)
Cylindrical coordinates (r, θ, z)		
$\frac{\partial\rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho r u_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho u_\theta) + \frac{\partial}{\partial z}(\rho u_z) = 0$		(1b)
Spherical coordinates (r, θ, ϕ)		
$\frac{\partial\rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r}(\rho r^2 u_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta}(\rho u_\theta \sin\theta) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \phi}(\rho u_\phi) = 0$		(1c)

Table 2: The Navier-Stokes equations for Newtonian fluids of constant ρ and μ

$\frac{\partial\vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho} \nabla P + \vec{g} + \nu(\nabla^2\vec{u})$		(2)
Rectangular coordinates (x, y, z)		
x-component	$\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + g_x + \nu \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2} \right)$	(2a)
y-component	$\frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + g_y + \nu \left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2} \right)$	(2b)
z-component	$\frac{\partial u_z}{\partial t} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + g_z + \nu \left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} \right)$	(2c)

04-CHEM-B1, Transport Phenomena

Table 2: Continued

Cylindrical coordinates (r, θ, z)

$$\begin{aligned} \text{r-component} \quad & \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r} \\ & = -\frac{1}{\rho} \frac{\partial P}{\partial r} + g_r + \nu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (ru_r)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2} \right] \end{aligned} \quad (2d)$$

$$\begin{aligned} \text{\theta-component} \quad & \frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + u_z \frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r} \\ & = -\frac{1}{\rho r} \frac{\partial P}{\partial \theta} + g_\theta + \nu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (ru_\theta)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2} \right] \end{aligned} \quad (2e)$$

$$\begin{aligned} \text{z-component} \quad & \frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} \\ & = -\frac{1}{\rho} \frac{\partial P}{\partial z} + g_z + \nu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right] \end{aligned} \quad (2f)$$

Spherical coordinates (r, θ, ϕ)

$$\begin{aligned} \text{r-component} \quad & \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + \left(\frac{u_\phi}{r \sin \theta} \right) \frac{\partial u_r}{\partial \phi} - \frac{u_\theta^2}{r} - \frac{u_\phi^2}{r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} + g_r \\ & + \nu \left[\frac{1}{r^2} \frac{\partial^2}{\partial r^2} (r^2 u_r) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial u_r}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_r}{\partial \phi^2} \right] \end{aligned} \quad (2g)$$

$$\begin{aligned} \text{\theta-component} \quad & \frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \left(\frac{u_\phi}{r \sin \theta} \right) \frac{\partial u_\theta}{\partial \phi} + \frac{u_r u_\theta}{r} - \frac{u_\phi^2}{r} \cot \theta = -\frac{1}{\rho r} \frac{\partial P}{\partial \theta} + g_\theta \\ & + \nu \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u_\theta}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (u_\theta \sin \theta) \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_\theta}{\partial \phi^2} \right. \\ & \left. + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} - \frac{2 \cot \theta}{r^2 \sin \theta} \frac{\partial u_\phi}{\partial \phi} \right] \end{aligned} \quad (2h)$$

$$\begin{aligned} \text{\phi-component} \quad & \frac{\partial u_\phi}{\partial t} + u_r \frac{\partial u_\phi}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\phi}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} + \frac{u_r u_\phi}{r} + \frac{u_\theta u_\phi}{r} \cot \theta = -\frac{1}{\rho r \sin \theta} \frac{\partial P}{\partial \phi} \\ & + g_\phi + \nu \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u_\phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (u_\phi \sin \theta) \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_\phi}{\partial \phi^2} \right. \\ & \left. + \frac{2}{r^2 \sin \theta} \frac{\partial u_r}{\partial \phi} + \frac{2 \cot \theta}{r^2 \sin \theta} \frac{\partial u_\theta}{\partial \phi} \right] \end{aligned} \quad (2i)$$

04-CHEM-B1, Transport Phenomena

Table 3: Shear stress-velocity gradient relationships for a constant viscosity Newtonian fluid

$[\hat{\tau}] = -\mu \left[\nabla \hat{u} + (\nabla \hat{u})^T \right] + \frac{2}{3} \nabla \cdot \bar{u} \quad (3)$	
Cartesian coordinates (x, y, z)	
$\tau_{xx} = -\mu \left[2 \frac{\partial u_x}{\partial x} \right] + \frac{2}{3} \nabla \cdot \bar{u} \quad (3a)$	(3a)
$\tau_{yy} = -\mu \left[2 \frac{\partial u_y}{\partial y} \right] + \frac{2}{3} \nabla \cdot \bar{u} \quad (3b)$	(3b)
$\tau_{zz} = -\mu \left[2 \frac{\partial u_z}{\partial z} \right] + \frac{2}{3} \nabla \cdot \bar{u} \quad (3c)$	(3c)
$\tau_{xy} = -\mu \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) = \tau_{yx} \quad (3d)$	(3d)
$\tau_{yz} = -\mu \left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} \right) = \tau_{zy} \quad (3e)$	(3e)
$\tau_{zx} = -\mu \left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right) = \tau_{xz} \quad (3f)$	(3f)
Cylindrical coordinates (r, θ, z)	
$\tau_{rr} = -\mu \left[2 \frac{\partial u_r}{\partial r} \right] + \frac{2}{3} \nabla \cdot \bar{u} \quad (3g)$	(3g)
$\tau_{\theta\theta} = -2\mu \left[\frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} \right] + \frac{2}{3} \mu \nabla \cdot \bar{u} \quad (3h)$	(3h)
$\tau_{zz} = -\mu \left[2 \frac{\partial u_z}{\partial z} \right] + \frac{2}{3} \mu \nabla \cdot \bar{u} \quad (3i)$	(3i)
$\tau_{r\theta} = -\mu \left[r \frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) + \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right] = \tau_{\theta r} \quad (3j)$	(3j)
$\tau_{\theta z} = -\mu \left[\frac{\partial u_\theta}{\partial z} + \frac{1}{r} \frac{\partial u_z}{\partial \theta} \right] = \tau_{z\theta} \quad (3k)$	(3k)
$\tau_{\theta z} = -\mu \left[\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right] = \tau_{z\theta} \quad (3l)$	(3l)

04-CHEM-B1, Transport Phenomena

Spherical coordinates (r, θ, ϕ)	
$\tau_{rr} = -\mu \left[2 \frac{\partial u_r}{\partial r} \right] + \frac{2}{3} \nabla \cdot \vec{u}$	(3m)
$\tau_{\theta\theta} = -2\mu \left[\frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} \right] + \frac{2}{3} \mu \nabla \cdot \vec{u}$	(3n)
$\tau_{\phi\phi} = -2\mu \left[\frac{1}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} + \frac{u_r}{r} + (u_\theta/r) \cot \theta \right] + \frac{2}{3} \mu \nabla \cdot \vec{u}$	(3o)
$\tau_{r\theta} = -\mu \left[r \frac{\partial}{\partial r} (u_\theta/r) + \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right] = \tau_{\theta r}$	(3p)
$\tau_{\theta\phi} = -\mu \left[\frac{\sin \theta}{r} \left[\frac{\partial}{\partial \theta} \left(\frac{u_\phi}{\sin \theta} \right) \right] + \frac{1}{r \sin \theta} \frac{\partial u_\theta}{\partial \phi} \right] = \tau_{\phi\theta}$	(3q)
$\tau_{r\phi} = -\mu \left[\frac{1}{r \sin \theta} \frac{\partial u_r}{\partial \phi} + r \frac{\partial}{\partial r} (u_\phi/r) \right] = \tau_{\phi r}$	(3r)

Table 4: The Energy Equation for Incompressible media

$\frac{\partial(\rho c_p T)}{\partial t} + (\vec{u} \cdot \nabla)(\rho c_p T) = [\nabla \cdot \alpha \nabla(\rho c_p T)] + \dot{T}_G$		(4)
Rectangular coordinates (x, y, z)		
$\frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z} = \frac{\partial}{\partial x} \left(\alpha \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\alpha \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\alpha \frac{\partial T}{\partial z} \right) + \frac{\dot{T}_G}{\rho c_p}$		(4a)
Cylindrical coordinates (r, θ, z)		
$\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r} \frac{\partial T}{\partial \theta} + u_z \frac{\partial T}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \alpha \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\alpha \frac{\partial T}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(\alpha \frac{\partial T}{\partial z} \right) + \frac{\dot{T}_G}{\rho c_p}$		(4b)
Spherical coordinates (r, θ, ϕ)		
$\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \alpha \frac{\partial T}{\partial r} \right)$		(4c)
$+ \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\alpha \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left(\alpha \frac{\partial T}{\partial \phi} \right) + \frac{\dot{T}_G}{\rho c_p}$		

04-CHEM-B1, Transport Phenomena

Table 5: The continuity equation for species A

$\frac{\partial C_A}{\partial t} + (\vec{u} \cdot \nabla)C_A = D_A \nabla^2 C_A + \dot{R}_{A,G} \quad (5)$
<p>Rectangular coordinates (x, y, z)</p> $\frac{\partial C_A}{\partial t} + u_x \frac{\partial C_A}{\partial x} + u_y \frac{\partial C_A}{\partial y} + u_z \frac{\partial C_A}{\partial z} = \frac{\partial}{\partial x} \left(D \frac{\partial C_A}{\partial x} \right) + \frac{\partial}{\partial y} \left(D \frac{\partial C_A}{\partial y} \right) + \frac{\partial}{\partial z} \left(D \frac{\partial C_A}{\partial z} \right) + \dot{R}_{A,G} \quad (5a)$
<p>Incompressible media, Cylindrical coordinates (r, θ, z)</p> $\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta} + u_z \frac{\partial C_A}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left(r D \frac{\partial C_A}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(D \frac{\partial C_A}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(D \frac{\partial C_A}{\partial z} \right) + \dot{R}_{A,G} \quad (5b)$
<p>Incompressible media, Spherical coordinates (r, θ, ϕ)</p> $\begin{aligned} \frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial C_A}{\partial \phi} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D \frac{\partial C_A}{\partial r} \right) \\ + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(D \sin \theta \frac{\partial C_A}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left(D \frac{\partial C_A}{\partial \phi} \right) + \dot{R}_{A,G} \end{aligned} \quad (5c)$