Professional Engineers Ontario

National Exams - December, 2013

07-Str-B3, Applications of Finite Elements

3 hours duration

Notes:

1. There are 4 pages in this examination, including the front page.
2. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. This is a closed book examination, with two 8½×11 in^2 pages of hand written notes.
4. Candidates may use one of the approved non-communicating calculators.
5. **Attempt to answer all three problems.**
6. All problems are of equal value.

Problem 1:

Figure 1 shows a tapered bar with an area $A(x)$ varying along the abscissa x. The problem of finding the displacement, strain and stress along the tapered bar in equilibrium is expressed as follows:

$$\begin{cases} \dfrac{du}{dx} = \dfrac{P}{EA(x)} \\ u(L) = 0 \\ EA(x)\dfrac{du}{dx}\Big|_{x=0} = P \end{cases}$$

Where $u(x)$ is the displacement field and P the axial force applied at the tip ($x = 0$).

Solve for the axial displacement and stress in the tapered bar shown in Figure 1.

1.1 using one constant-area element
1.2 using two constant-area elements

1.3 Compare the displacement and stress fields obtained by the finite solutions with the exact solution
1.4 Comment on these results

For each case, evaluate the area at the center of each element length.
Let $A_0 = 2\ in^2, A_1 = 3\ in^2, L = 20\ in, E = 10 \times 10^6\ psi$ and $P = 1000\ lb$.

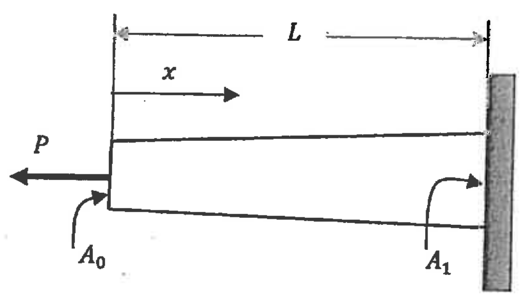

Figure 1

Problem 2

Q1. How do the stresses vary within a four-node rectangular element modelling in-plane stress problems, explain your answer.

Q2: the plane stress element only allows for in-plane displacements, while beam element resists displacements and rotations. How can we combine the plane stress and beam elements and still ensure compatibility.

Q3: Write in column 2 if you can use **plane strain** or **plane stress** to model the structure described in column1. If neither case applies write **"Neither"**

Column 1	Column 2
a flat slab floor of a building with vertical loading perpendicular to the slab	
a wall subjected to wind loading (the wall acts as a shear wall with loads in the plane of the wall)	
a tensile plate with a hole drilled transversally through it	
a concrete dam subjected to the hydrostatic pressure of the reservoir	
a soil mass subjected to a strip footing load	
a wrench subjected to a force in its plane	
a wrench subjected to a twisting forces (the twisting forces act out of the plane of the wrench)	
a triangular plate connection with loads in the plane of the triangle	

Problem 3

3.1 The first three shape functions associated to, the degrees of freedom v_1, ϕ_1 *and* v_2 , of a beam element in plane are given by (refer to Figure 3.1):

$$N_1(x) = \frac{1}{L^3}(2x^3 - 3x^2L + L^3), \qquad N_2(x) = \frac{1}{L^3}(x^3L - 2x^2L^2 + xL^3)$$

$$N_3(x) = \frac{1}{L^3}(-2x^3 + 3x^2L)$$

Calculate the fourth shape function $N_4(x)$ associated to the degree of freedom ϕ_2.

3.2 Using the work–equivalence method calculate the set of discrete loads to replace the linearly distributed force at the center of the beam shown in Figure 3.2.

3.3 The stiffness matrix of a beam element is given below; calculate the displacement and the slope at the center of the beam shown in Figure 3.2.

$$[k] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

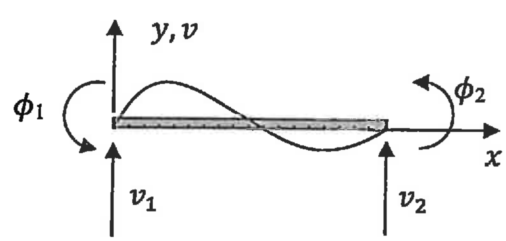

Figure 3.1

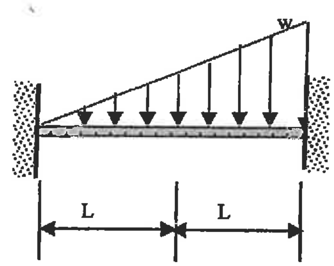

Figure 3.2