National Exams May 2012

98-Phys-B5, Control

3 hours duration

NOTES:

1. This is a CLOSED BOOK EXAM. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a single-sided, handwritten, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet has to be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining five. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. **PLEASE WRITE ANSWERS DIRECTLY IN THIS EXAM PAPER – DO NOT USE EXAM BOOKS.** If necessary, you may write on the backside of the pages as long as you write the final answers in the space indicated, and point to where the full calculations are.

YOUR MARKS		
QUESTIONS 1 AND 2 ARE COMPULSORY:		
Question 1	20	
Question 2	20	
CHOOSE THREE OUT OF THE REMAINING FIVE QUESTIONS:		
Question 3	20	
Question 4	20	
Question 5	20	
Question 6	20	
Question 7	20	
TOTAL:	100	

A Short Table of Laplace Transforms

Laplace Transform	Time Function
1	$\sigma(t)$
$\frac{1}{s}$	$1(t)$
$\frac{1}{s^2}$	$t \cdot 1(t)$
$\frac{1}{s^k}$	$\frac{t^k}{k!} \cdot 1(t)$
$\frac{1}{s+a}$	$e^{-at} \cdot 1(t)$
$\frac{1}{(s+a)^2}$	$t \cdot e^{-at} \cdot 1(t)$
$\frac{a}{s(s+a)}$	$(1-e^{-at}) \cdot 1(t)$
$\frac{a}{s^2+a^2}$	$\sin(at) \cdot 1(t)$
$\frac{s}{s^2+a^2}$	$\cos(at) \cdot 1(t)$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cdot \cos(bt) \cdot 1(t)$
$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \cdot \sin(bt) \cdot 1(t)$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$\left(1-e^{-at} \cdot \left(\cos bt + \frac{a}{b}\sin bt\right)\right) \cdot 1(t)$
$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \cdot \sin(\omega_n \sqrt{1-\xi^2} t) \cdot 1(t)$
$\frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$	$\left(1-\frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \cdot \sin(\omega_n \sqrt{1-\xi^2} t + \cos^{-1}\xi)\right) \cdot 1(t)$
$F(s) \cdot e^{-Ts}$	$f(t-T) \cdot 1(t)$
$F(s+a)$	$f(t) \cdot e^{-at} \cdot 1(t)$
$sF(s) - f(0+)$	$\frac{df(t)}{dt}$
$\frac{1}{s}F(s)$	$\int_{0+}^{+\infty} f(t)dt$

Question 1 (Compulsory)

Pole-Zero Map. Zero-Pole-Gain and Polynomial Ratio forms of a Transfer Function. Second Order Dominant Poles Model from Pole-Zero Locations. Second Order Model and its parameters. Step Response Specification and how they relate to the parameters of the Second Order Model. System Type, Error Constants and Steady State Errors.

Part A (10 marks)

A certain closed loop LTI system is described as having four poles and one zero, as follows: $p_1 = -6, p_2 = -1 + j6, p_3 = -1 - j6, p_4 = -14$ and $z_1 = -5.8$. It is also recorded that this closed loop system has a DC Gain of 0.95.

1) **(5 marks)** The system transfer function can be written in a Zero-Pole-Gain form, shown in Table Q1.1, where p_k are system poles, z_i are system zeros, and K is the so-called multiplier gain. Note the multiplier gain K in this form of the transfer function is NOT the same as the DC Gain, and it needs to be calculated based on the information provided. Write out the transfer function, $G_{cl}(s)$, of our LTI system in this form, and place it in Table Q1.1.

2) **(5 marks)** Use the space provided in Figure Q1.1 to draw the pole-zero map for the closed loop system and based on it, briefly explain why it is appropriate to model the closed loop system behaviour by using the standard second order system model. Next, derive the model parameters: the damping ratio, ζ, the frequency of natural oscillations, ω_n, and the DC Gain, K_{dc}, write out the model transfer function, $G_m(s)$, and place both the parameters and the model transfer function in Table Q1.1.

Question 1 Continued

Table Q1.1

<table>
<tr><td colspan="2">The exact closed loop system transfer function, $G_{cl}(s)$, in the Pole-Zero-Gain form, is as follows:
$$G_{cl}(s) = \frac{K \cdot \prod_{i=1}^{m}(s - z_i)}{\prod_{k=1}^{n}(s - p_k)}$$
$G_{cl}(s) =$ ____________________</td></tr>
<tr><td colspan="2">The closed loop transfer function $G_{cl}(s)$ can be approximated by this 2nd order model:
$$G_m(s) = K_{dc}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$
$G_m(s) =$ ____________________</td></tr>
<tr><td>Damping ratio is:</td><td>$\zeta =$</td></tr>
<tr><td>Frequency of natural oscillations is:</td><td>$\omega_n =$</td></tr>
<tr><td>Closed loop DC gain is:</td><td>$K_{dc} =$</td></tr>
</table>

Question 1 Continued

Question 1 Continued

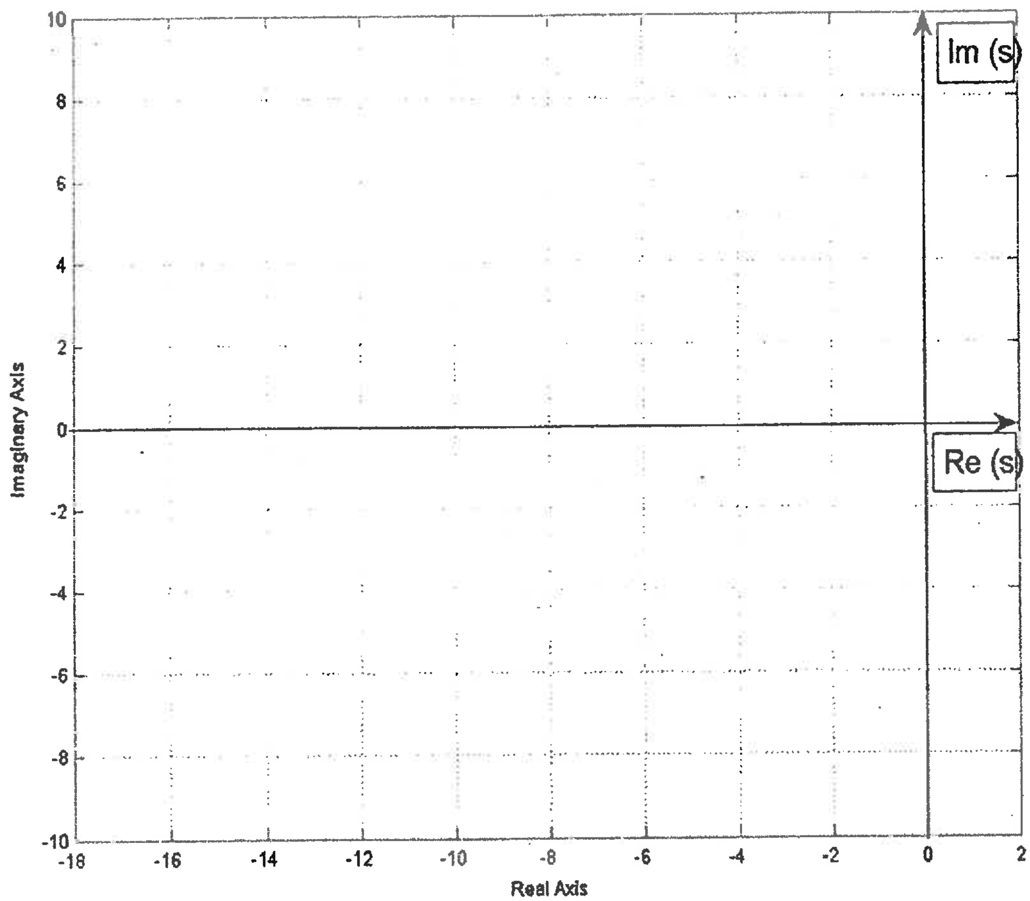

Figure Q1.1

Place your explanation, of why the 2nd order model is appropriate, here:

____

____

____

____

Question 1 Continued

Part B (10 marks)

Consider the following block diagram of a system under Proportional Control as shown in Figure Q1.2 below:

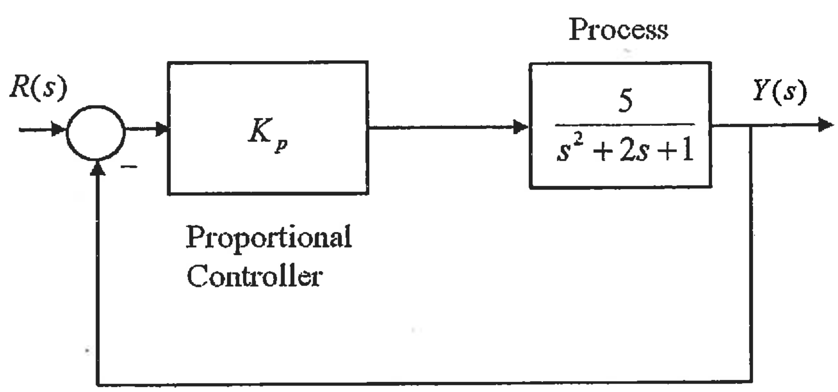

Figure Q1.2

1) **(3 marks)** Derive the closed loop transfer function, $G_{cl}(s) = \frac{Y(s)}{R(s)}$, in terms of the controller parameter, K_p. and write it in Table Q1.2. Next, consider the uncompensated system (i.e. with $K_p = 1$) and calculate the following parameters of the closed loop system: DC Gain, K_{dc}, frequency of natural oscillations, ω_n, and damping ratio, ζ. Place the parameters in Table Q1.2.

2) **(4 marks)** Estimate the following specifications for the closed loop step response of this system: Percent Overshoot, PO, Rise Time, $T_{rise(10-90\%)}$, Settling Time, $T_{settle(\pm 2\%)}$, and the frequency of actual oscillations, ω_d. Place your answers in Table Q1.2.

1) **(3 marks)** What is the System Type? Compute the system Error Constants and Steady State Errors: K_{pos}, $e_{ss(step\%)}$, K_v, $e_{ss(ramp)}$, K_a, $e_{ss(parab)}$. Place your answers in Table Q1.2.

Question 1 Continued

Question 1 Continued

Question 1 Continued

Table Q1.2

The closed loop transfer function $G_{cl}(s)$ of the system under Proportional Control is: $G_{cl}(s) = \rule{8cm}{0.4pt}$			
The closed loop parameters for the uncompensated system (i.e. with $K_p = 1$), are as follows:			
Damping ratio is:		$\zeta =$	
Frequency of natural oscillations is:		$\omega_n =$	
Closed loop DC gain is:		$K_{dc} =$	
$PO =$	$T_{rise(10-90\%)} =$	$T_{settle(\pm 2\%)} =$	$\omega_d =$
System Type:	$K_{pos} =$	$K_v =$	$K_a =$
N =	$e_{ss(step)\%} =$	$e_{ss(ramp)} =$	$e_{ss(parab)} =$

Question 2 (Compulsory)

Proportional Control (P) and Proportional + Integral + Derivative Control (PID). Closed Loop Characteristic Equation Shaping based on Dominant Second Order Model. Block Diagram Reduction, Time Response Specifications, Steady State Errors, Error Constants & System Type.

Part A - Proportional Control (6 marks)

Consider again the control system in Question 1, under proportional Control, as shown in Figure Q1.2.

1. **(2 marks)** Refer to Table Q1.2 for the closed loop transfer function, $G_{cl}(s) = \frac{Y(s)}{R(s)}$, in terms of the controller parameter, K_p, and calculate the Proportional Gain required to have the Steady State Error of the closed loop step response, $e_{ss(step\%)}$, equal to 5%. Place that value in Table Q2.1.
2. **(2 marks)** What would be the resulting Percent Overshoot, PO, and the Settling Time (within 2% of the steady state value), $T_{settle(\pm 2\%)}$, of the closed loop step response, at that value of the Proportional Gain? Place your answers in Table Q2.1.
3. **(2 marks)** Is it possible to reduce the Steady State Error in this system step response to zero using Proportional Control? Briefly discuss below.

Question 2 Continued

Part B - Proportional + Integral + Derivative (PID) Control (14 marks)

Now consider the same system where the Controller scheme was modified to incorporated a PID Control. As shown in Figure Q2.1 below, it is implemented by placing a Proportional + Integral (PI) Controller in the forward path, and a Rate Feedback (Derivative) in an inner feedback loop.

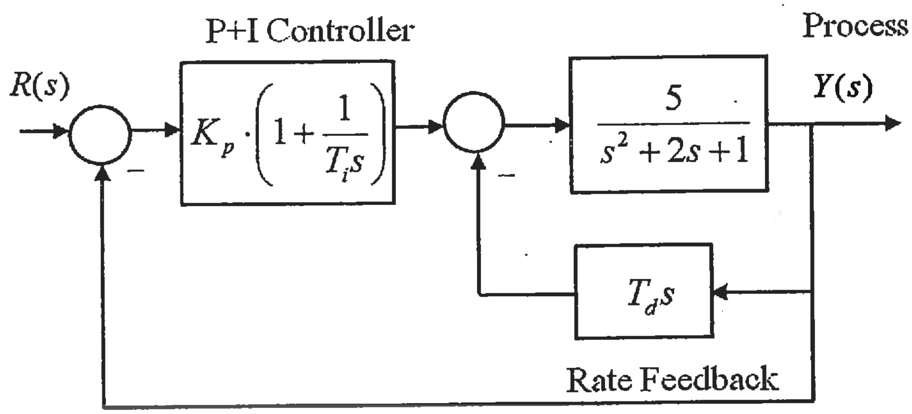

Figure Q2.1

1. **(5 marks)** Derive the closed loop transfer function, $G_{cl}(s) = \frac{Y(s)}{R(s)}$, in terms of the controller parameters, K_p, T_i and T_d. Place the closed loop transfer function expression in Table Q2.1.

2. **(9 marks)** It is desired that the closed loop system step response has a zero Steady State Error, $e_{ss(step\%)}$, Percent Overshoot, PO, of 10% and the settling time (±2% criterion), $T_{settle(\pm 2\%)}$, of 1 second. Compute appropriate controller parameters and place their values in Table Q2.1.

   HINT: This is a third order system. Assume that the closed loop system has two dominant poles with a damping ratio, ζ, and the natural frequency, ω_n, that correspond to the desired step response specifications, and that the third closed loop pole is at a location ten (10) times further to the left of the s-plane than the Real Part of the dominant poles.

Question 2 Continued

Question 2 Continued

Question 2 Continued

Table Q2.1

Part A - Proportional Control	
Proportional Controller Gain to get $e_{ss(step\%)} = 5\%$ is: $K_p =$	
At this value of the Proportional Gain, the Percent Overshoot, PO, and the Settling Time, $T_{settle(\pm 2\%)}$, will be:	
$PO =$	$T_{settle(\pm 2\%)} =$

Part B - Proportional + Integral + Derivative (PID) Control		
The closed loop transfer function of this system, in terms of PID Controller parameters, is written as: $G_{cl}(s) = \underline{\hspace{6cm}}$		
PID Controller parameters are computed as follows:		
$K_p =$	$T_i =$	$T_d =$

Question 3

Closed Loop Stability, determining the range of safe operations under Proportional Control - Routh-Hurwitz and Bode Criteria. Steady State Errors, Error Constants and System Type from Frequency Response.

Consider a certain control system under Proportional Control, as shown in Figure Q3.1. Open loop frequency response plots of the system, with Controller Gain K_p=1, are shown in Figure Q3.2.

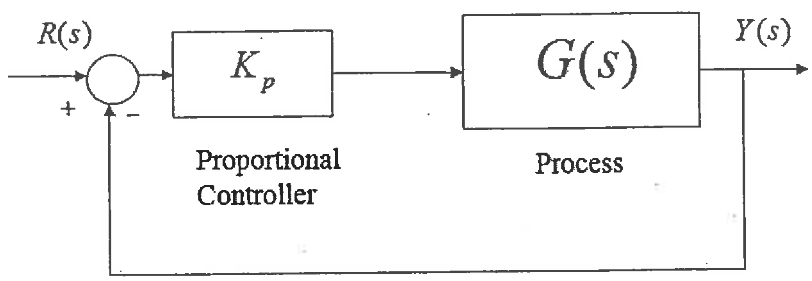

Figure Q3.1

1) **(6 marks)** Find the system Gain Margin, Phase Margin and corresponding crossover frequencies. Determine the critical value of the gain, K_{crit} , at which the system becomes marginally stable, and the corresponding frequency of marginally stable oscillations, ω_{osc}. Determine the range of gains K_p to provide a stable closed loop system response and place your answers in Table Q3.1.

2) **(14 marks)** The process transfer function, G(s), is known, and given as follows:

$$G(s) = \frac{20(s+1)}{s(s^2+10s+5)(s+2)}$$

Verify the results of Item 1) by applying the Routh-Hurwitz Criterion of Stability, i.e. find the critical value of the gain, K_{crit} , at which the system becomes marginally stable, and the corresponding frequency of marginally stable oscillations, ω_{osc}. How do they compare to the values in Item 1)? Place your answers in Table Q3.1.

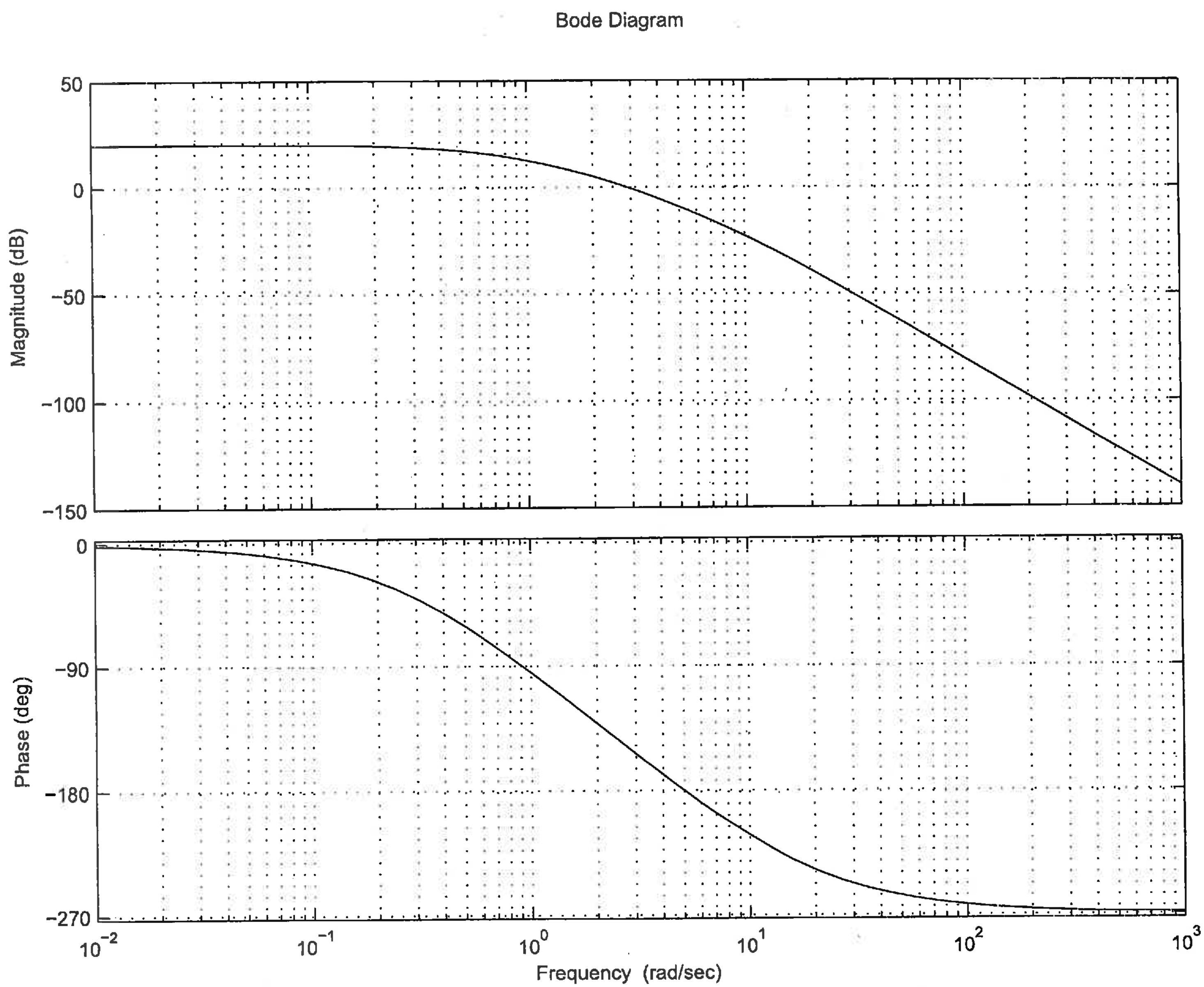
Bode Diagram
50
0
-50
-100
-150
Magnitude (dB)
0
-90
-180
-270
Phase (deg)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec)

Figure Q3.2

Question 3 Continued

Use this page to do your analytical Routh-Hurwitz calculations.

Question 3 Continued

Table Q3.1

Gain Margin	Corresponding crossover frequency	Phase Margin	Corresponding crossover frequency	Critical gain from Bode plot	Frequency of oscillations from Bode plot
$G_{m\,dB} =$	$\omega_{cg} =$	$\Phi_m =$	$\omega_{cp} =$	$K_{crit} =$	$\omega_{osc} =$

Range of gains for stable closed loop operation from Bode plot: $< K_p <$

Critical gain From Routh-Hurwitz	$K_{crit} =$	Frequency of oscillations from Routh-Hurwitz	$\omega_{osc} =$

Range of gains for stable closed loop operation from Routh-Hurwitz Criterion: $< K_p <$

Question 4

Root Locus Analysis, Second Order Dominant Poles Model, Step Response Specifications, Proportional Controller Design.

Consider a certain closed loop control system under Proportional Control shown in Figure Q4.1:

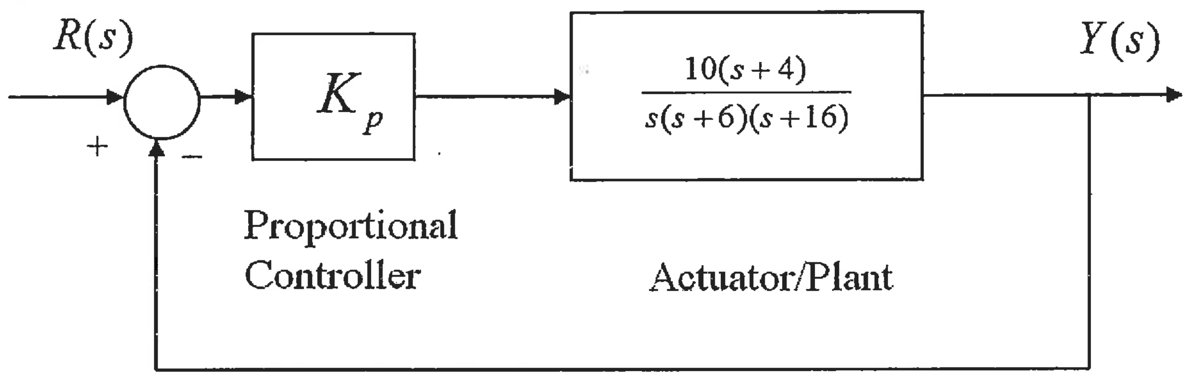

Figure Q4.1

1) **(8 marks)** In the space provided in Figure Q4.2, sketch a detailed Root Locus for the system, including crossovers with the imaginary axis, if any, break-away/break-in coordinates, if any, asymptotes, if any, a centroid, etc. If you are using estimates, explain why. Place the Root Locus parameters in Table Q4.1.

2) **(4 marks)** Assume that the closed loop system can be approximated by a second order model. Determine the appropriate value of the closed loop system equivalent damping ratio ζ so that the closed loop system step response displays a Percent Overshoot of approximately 5%. Next, use this information and the Root Locus sketch from Item 1) to determine the remaining second order model parameters K_{dc} and ω_n, and calculate the model transfer function $G_{model}(s)$. Place your answers in Table Q4.2.

3) **(6 marks)** Based on the Root Locus in Item 1) and on the results of Item 2), calculate the required value of the Proportional Gain K_p to achieve the desired closed loop system equivalent damping ratio ζ and find the actual closed loop transfer function at the calculated Gain, $G_{closed}(s)$. Place your answers in Table Q4.2.

3) **(2 marks)** Next, compare $G_{closed}(s)$ and $G_{model}(s)$. Does the model represent adequately the dominant dynamic of the actual closed loop system? Comment briefly it in the lined space provided.

Table Q4.1

Root Locus centroid is at:	$\sigma =$
Root Locus asymptotic angles are equal to:	$\theta_i =$
The critical value of the Proportional Gain for which the system is marginally stable, and the frequency of resulting oscillations:	$K_{crit} =$ $\omega_{osc} =$
Break-away (leave blank if not applicable) coordinate is at:	$s_b =$
Break-in (leave blank if not applicable) coordinate is at:	$s_b =$

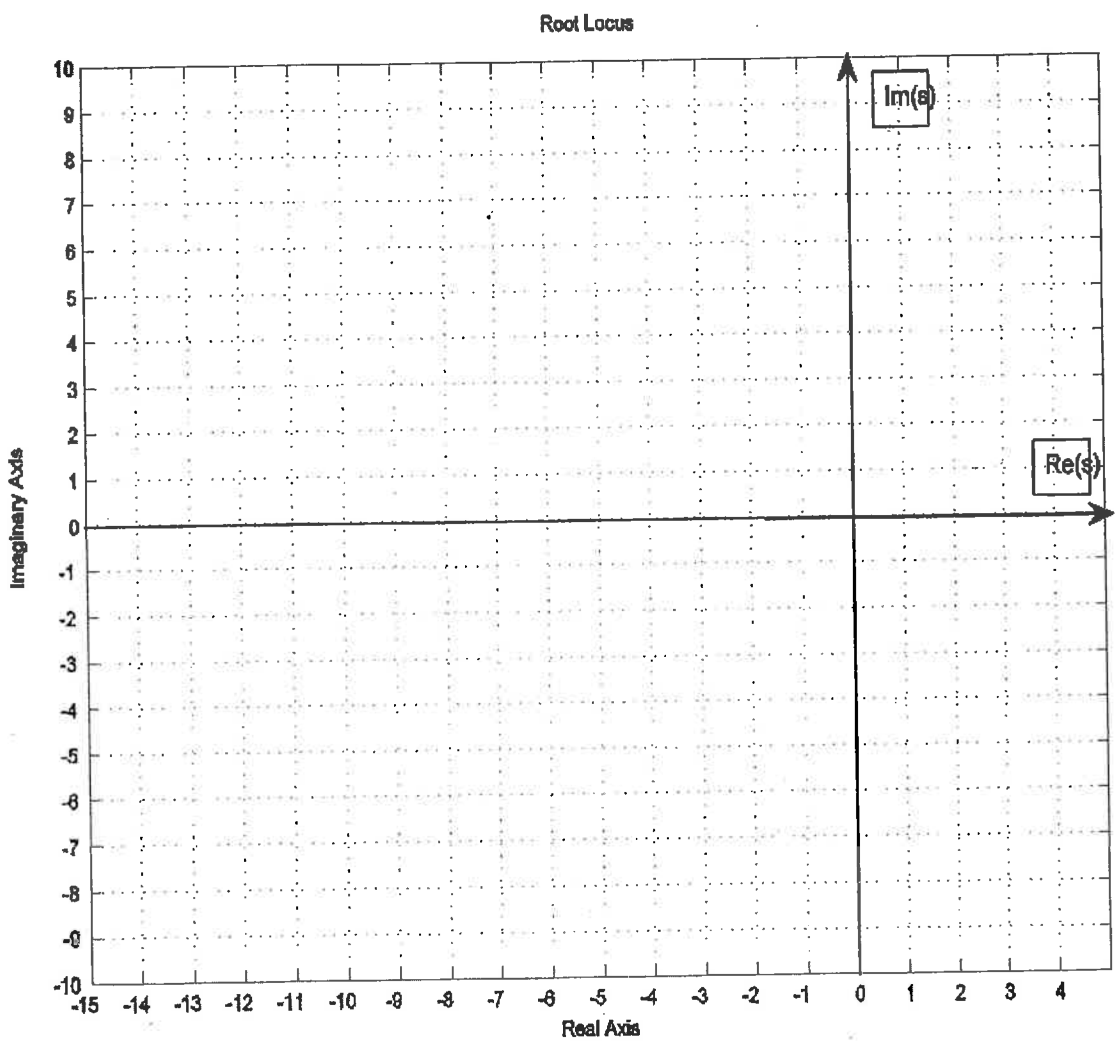

Figure Q4.2

Question 4 Continued

Question 4 Continued

Question 4 Continued

Question 4 Continued

Table Q4.2

$\zeta =$	$\omega_n =$	$K_{dc} =$
Assumed second order model: $G_{model}(s) =$		
Controller gain required for $PO = 5\%$ is equal to:		$K_p =$
Closed loop transfer function at this value of K_p: $G_{closed}(s) =$		

Question 5

State Space Model from Transfer Functions, System Stability, Pole Placement by State Feedback Method, Steady State Errors to Step and Ramp Inputs.

Consider a linear process described by the signal flow graph in Figure Q5.1:

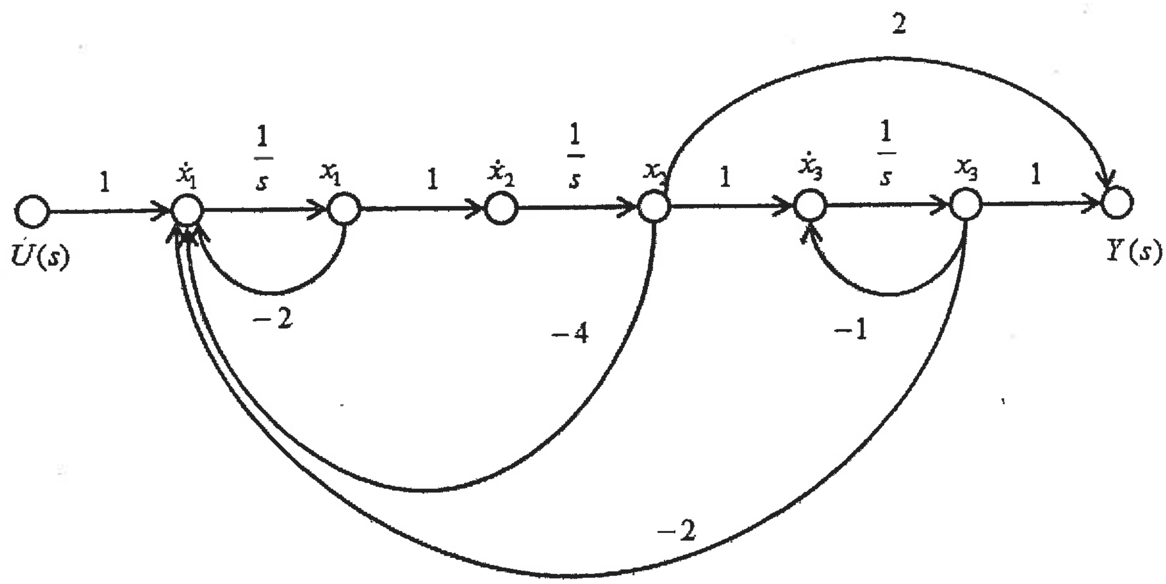

Figure Q5.1

1) **(6 marks)** Derive a set of the corresponding state equations - follow the choice of state variables as indicated in Figure Q5.1.

2) **(6 marks)** Determine if the process is stable.

3) **(8 marks)** A control system is to be built around the process by utilizing a state-variable feedback according to the following equation:

$$u = K \cdot \left(r - \mathbf{k}^T \cdot \mathbf{x}\right)$$

Determine the values of the gain constant K and the state feedback vector $\mathbf{k}$ so that the closed loop system will have poles at: -10 and $-1 \pm j$, and the steady-state error to a step input is to be zero.

Question 5 Continued

Question 5 Continued

Question 5 Continued

Question 6

Controller Design in Frequency Domain, Second Order Dominant Poles Model from Frequency Response Plots, Step Response Specifications and Steady State Errors.

Consider a unit feedback closed loop control system, as shown in Figure Q6.1. The system is to operate under **Lag Control.** The Lag Controller transfer function is shown below:

$$G_c(s) = K_{dc} \cdot \frac{1 + T\alpha s}{1 + Ts}$$

where T is the so-called Lag Time Constant and $\alpha < 1$. The Plant transfer function G(s) is as follows:

$$G(s) = \frac{30(s+2)}{(s+0.1)^2(s+20)^2}$$

The closed loop performance requirements are:

- The Steady State Error for the unit step input for the compensated closed loop system is one half of the Steady State Error for the uncompensated system.
- Percent Overshoot is approximately 10%.

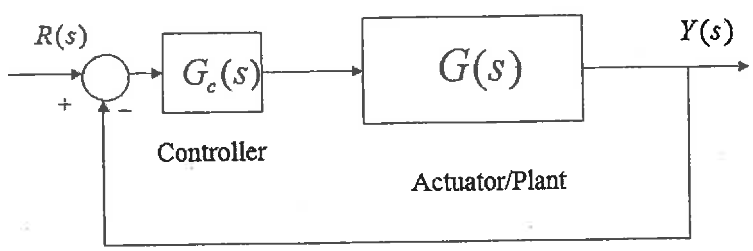

Figure Q6.1

1) **(5 marks)** Open loop frequency response plots of G(s) are shown in Figure Q6.2. Check what the current values of the Phase margin, Φ_m, and the Crossover Frequency, ω_{cp} are and place your answers in Table Q6.1. Next, estimate the uncompensated closed loop step response specs: Percent Overshoot, PO, Steady State Error, $e_{ss(step\%)}$, and Settling Time, $T_{settle(\pm 2\%)}$. Place the specs values in Table Q6.1 as well.

2) **(3 marks)** Next, based on the specifications, calculate the required values of the Phase Margin for the compensated system, Φ_{mc}, and the DC gain of the controller, K_{dc}, and also place them in Table Q6.1.

3) **(10 marks)** Design the Lag Controller such that it meets the closed loop response requirements and write the Lag Controller transfer function and its parameters in Table Q6.2.

4) **(2 marks)** For your Controller, estimate the compensated closed loop step response specs: Percent Overshoot, PO, Steady State Error, $e_{ss(step\%)}$, and Settling Time, $T_{settle(\pm 2\%)}$ and place the specs values in Table Q6.2.

Table Q6.1

Phase Margin of the uncompensated system is:		$\Phi_{m(uncomp)} =$
Crossover Frequency for the Phase Margin of the uncompensated system is:		$\omega_{cp(uncomp)} =$
DC Gain of the uncompensated system is:		$K_{dc(uncomp)} =$
DC Gain		
Uncompensated Step Response Specifications are:		
$PO =$	$e_{ss(step\%)} =$	$T_{settle(\pm 2\%)} =$
Phase Margin of the compensated system should be:		$\Phi_{m(comp)} =$
DC Gain of the compensated system should be:		$K_{dc(comp)} =$

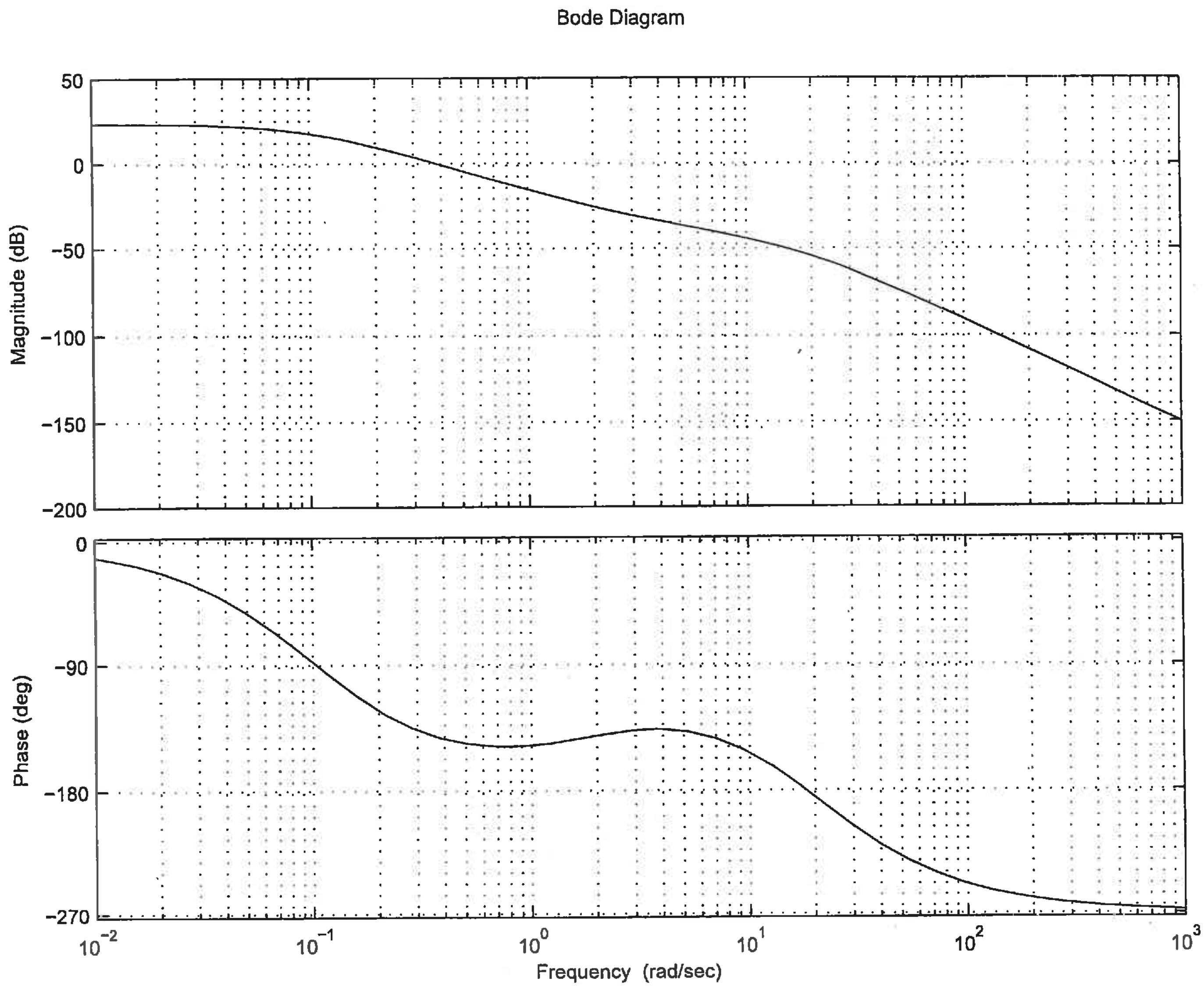
Bode Diagram
Magnitude (dB)
50
0
-50
-100
-150
-200
Phase (deg)
0
-90
-180
-270
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec)

Figure Q6.2

Question 6 Continued

Question 6 Continued

Question 6 Continued

Table Q6.2

Lag Controller transfer function is: $G_c(s) = $ ____________________ Where the Controller parameters are:		
$K_{dc} =$	$T =$	$\alpha =$
Compensated Step Response Specifications are:		
$PO =$	$e_{ss(step\%)} =$	$T_{settle(\pm 2\%)} =$

Question 7

Frequency Domain: Polar Plots and Nyquist Stability Criterion. Second Order Dominant Poles Model from Frequency Response Plots, Step Response and Specifications. Steady State Errors. Analytical solution for Impulse and Step Response, Laplace Transform Tables.

Part A (4 marks)

A certain open loop control system has **three poles** on the left-hand side of the s-plane (i.e. in LHP), and **one pole** on the right-hand side (i.e. in RHP). The corresponding Nyquist diagram was generated using a clockwise (CW) Γ path in the s-plane, and is shown in Figure Q7.1 below.

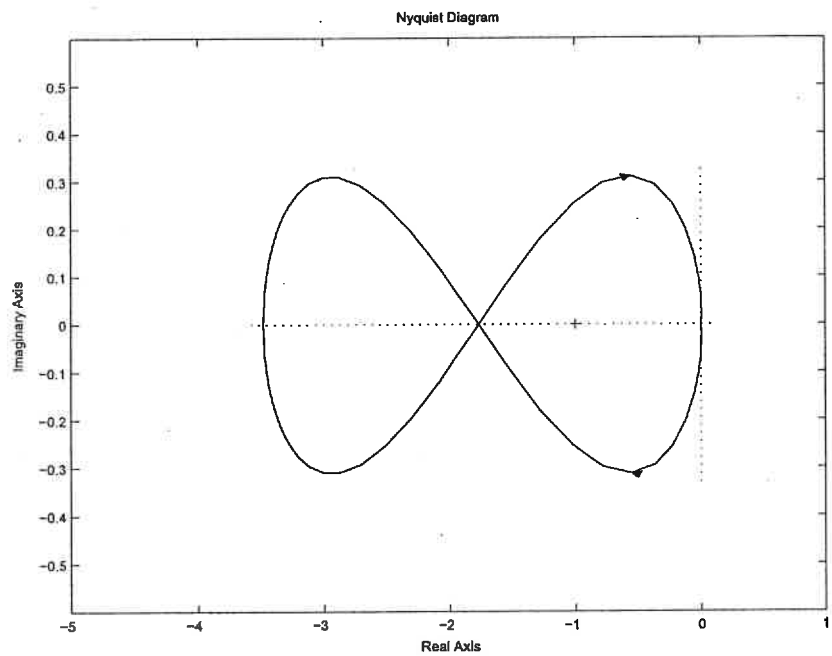

Figure Q7.1

1) Apply the Nyquist criterion of stability to determine if the system is stable;
2) How many closed loop poles are in LHP, and how many, if any, are in the RHP?
3) Calculate the system Gain Margin, as a V/V ratio as well as in decibels.

Write your answers in Table Q7.1:

Table Q7.1

System Gain Margin is:		Is the system stable? Check YES or NO below:		Number of closed loop poles in LHP	Number of closed loop poles in RHP
V/V	dB	**YES**	**NO**		

Part B (4 marks)

Consider the following transfer function of a certain process G(s):

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s^2 + 3s + 2}{s^3 + 2s^2 + 5s + 3}$$

1) In the space provided in Figure Q7.2 below, complete a signal flow graph diagram so that it will represent G(s). Note that there are several different possible realizations of this transfer function.

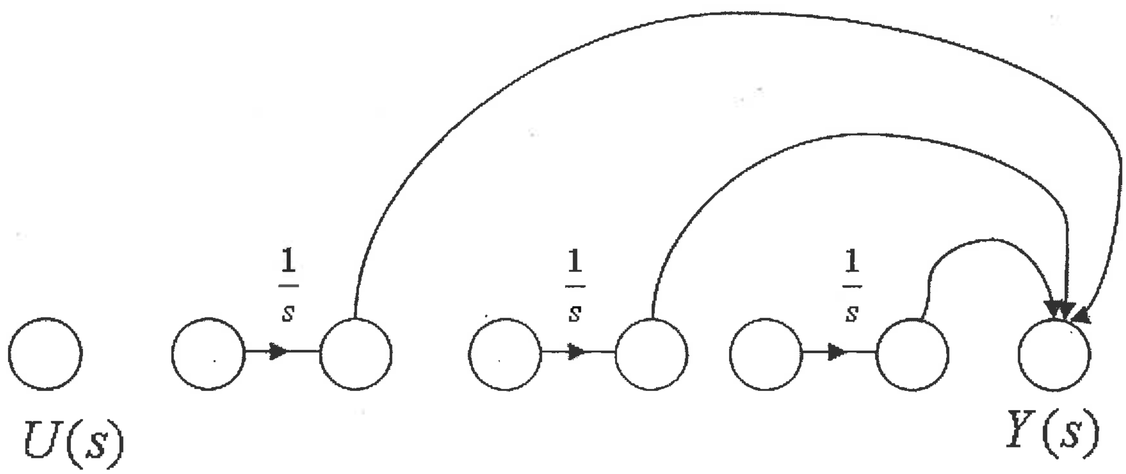

Figure Q7.2: Complete the Signal Flow Graph here.

Part C (4 marks)

Consider four separate second order systems, with pole pair maps shown in Figure Q7.3. Choose answers to the following questions by putting a checkmark in the appropriate box. Checking off more than one box invalidates the answer.

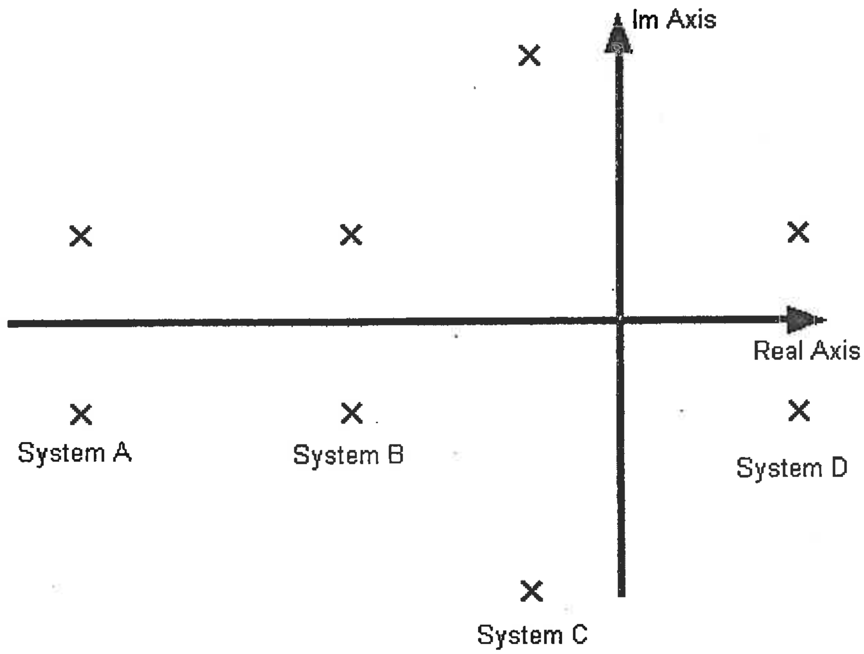

Figure Q7.3

	System A	**System B**	**System C**	**System D**
1) Which of the four systems exhibits the <u>largest</u> percent overshoot (PO) in its step response?	☐	☐	☐	☐
2) Which of the four systems is <u>unstable</u>?	☐	☐	☐	☐
3) Which of the four systems exhibits the <u>shortest</u> settling time in its step response?	☐	☐	☐	☐
4) Which of the four systems has the <u>highest</u> frequency of oscillations in its step response?	☐	☐	☐	☐

Part D (8 marks)

The input-output relationship in a certain control system is described by the following ordinary differential equation:

$$\ddot{y}(t) + 12\dot{y}(t) + 32y(t) = 20\dot{u}(t) + 40u(t)$$

where *u(t)* is the system input and *y(t)* is the system output.

1) Find the system transfer function $G(s) = \frac{Y(s)}{U(s)}$

2) Find the function *y(t)*, $t \geq 0$, describing the system response to a unit step input, *u(t)* = *1(t)*.

Question 7 Continued