

National Exams May 2012

07-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Candidates may use a Casio or Sharp approved calculator. This is a **closed book** exam. No aids other than semi-log graph papers are permitted.
3. Any four questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Question 1**Given:**

$$G(s) = \frac{K(s^2 + 0.5s + 100)}{S(s + 1)(s + 2)}$$

- Sketch the root Locus
- If the Loci cross the imaginary axis, determine values of w at the axis intersections.
- If we want complex roots with $\zeta = 0.707$, what must be the value of K ?

Question 2

- The characteristic equation for a feedback control system is:

$$(s + 2)(s^2 + 4s + 8) + K = 0$$

Determine the range of values of K for which the system is stable.

- Use the Laplace transform method to solve the following differential equations.

- $\frac{dy}{dt} + y = 2 \sin t$

- $\frac{dy}{dt} + y = 2 \cos t$

All the initial conditions are zero.

Question 3

Comment on the stability with the closed-loop transfer function given by each of the following.

$$\text{a) } T(s) = \frac{5(s+2)}{(s+1)(s^2+s+1)}$$

$$\text{b) } T(s) = \frac{5}{(s-1)(s^2+s+1)}$$

$$\text{c) } T(s) = \frac{(s^2+3)}{(s+1)(s^2+s+1)}$$

Question 4

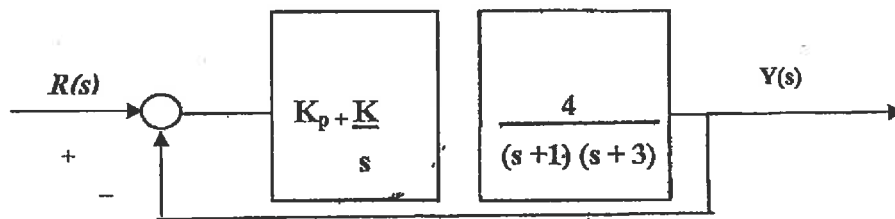
Given the transfer function

$$G(s) = \frac{10(s+1)}{s(s^2+20\zeta s+100)}$$

- Sketch the Bode diagram for $\zeta = 1$.
- Repeat (a) for $\zeta = 0.1$.

Question 5

Consider the control system shown in the figure below with a PI controller. Determine the values of K_p and K_i such that when $r(t) = t$, i.e., a unit ramp function, the steady-state error is 0.03.



Question 6

The block diagram of a hydraulic machining control system is shown below. Replace the negative feedback with positive feedback. Show that the characteristic equation can be represented in the form:

$$a_4s^4 + a_3s^3 + a_2s^2 + s - K = 0$$

where a_4 , a_3 , and a_2 are positive constants and are related to the system parameters. Using the Routh-Hurwitz criterion, investigate the stability for $K > 0$. Comment on the effect of replacing negative feedback by positive feedback.

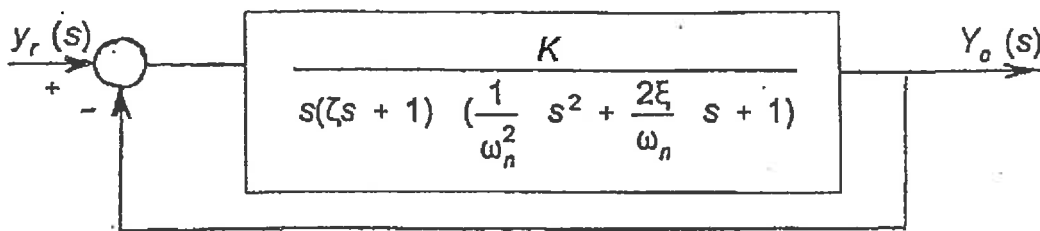
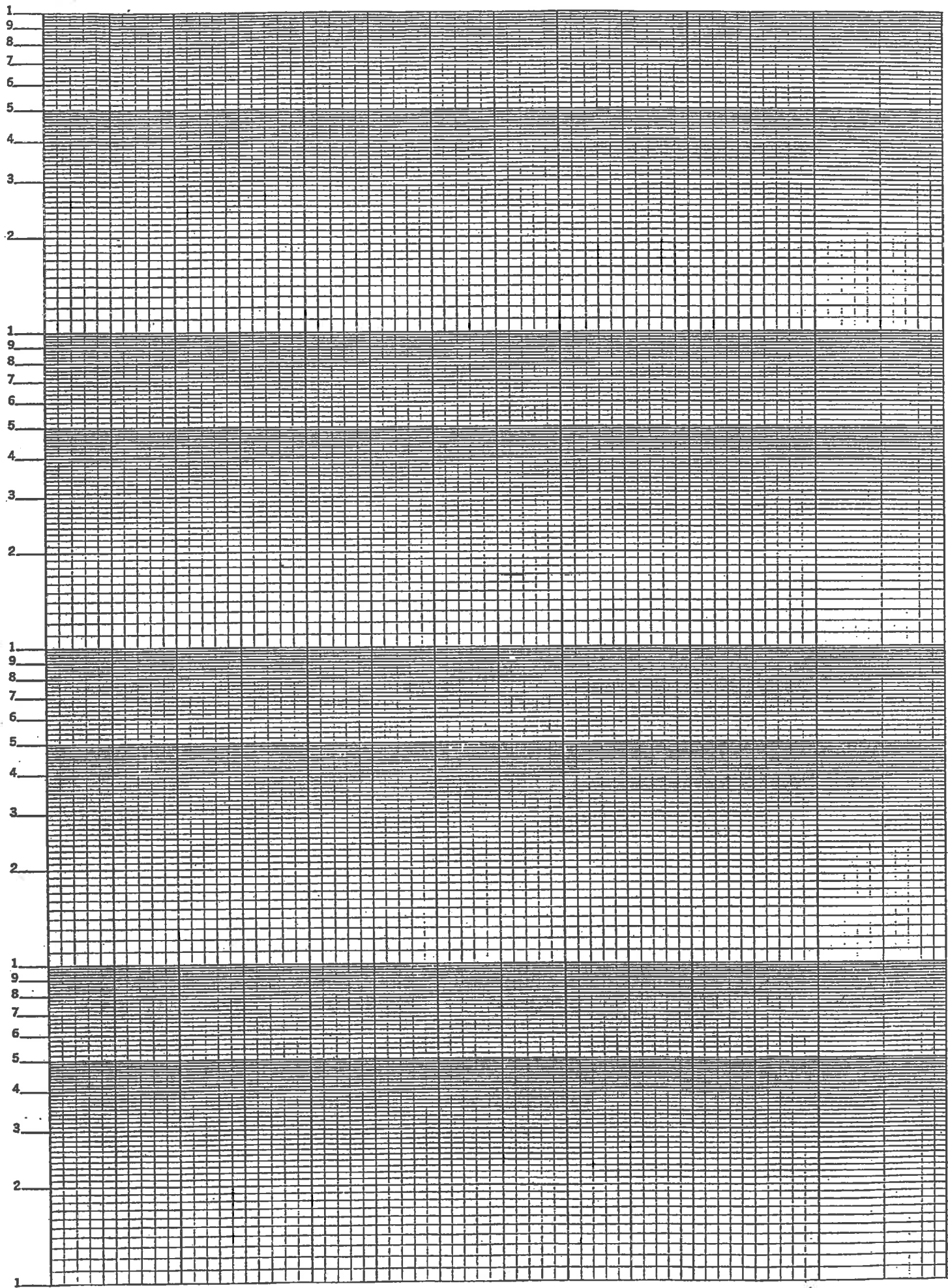


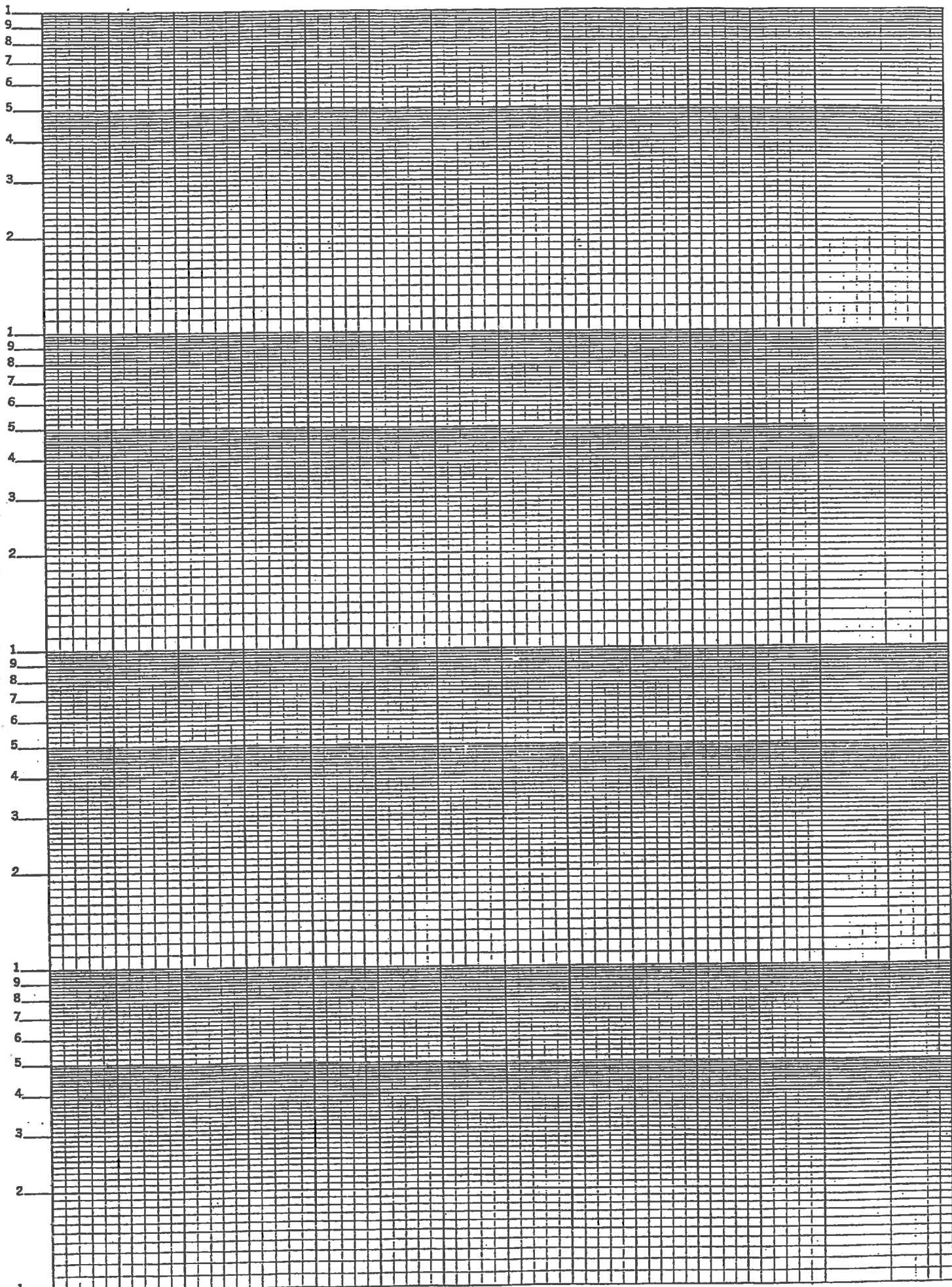
TABLE 6-1
LAPLACE TRANSFORM PAIRS

TABLE 6-1 (Continued)

Functions of Time $f(t)$ for $0 \leq t$	Laplace Transform $\mathcal{L}\{f(t)\}$	Functions of Time $f(t)$ for $0 \leq t$	Laplace Transform $\mathcal{L}\{f(t)\}$
1. $f(t)$	$\int_0^\infty f(t)e^{-st} dt = F(s)$	19. $t \sin at$	$\frac{2as}{(s^2 + a^2)^2}$
2. $x(t) + y(t)$	$X(s) + Y(s)$	20. $t \cos at$	$\frac{s^2 - a^2}{(s^2 + a^2)^2}$
3. $Kf(t)$	$KF(s)$	21. $e^{-at} \sin at$	$\frac{a}{(s+a)^2 + a^2}$
4. $\frac{df(t)}{dt}$	$sF(s) - f(0)$	22. $e^{-at} \cos at$	$\frac{s+a}{(s+a)^2 + a^2}$
5. $\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0) - \frac{df(0)}{dt}$	23. $e^{-\zeta\omega_n t} \sin \omega_n(1 - \zeta^2)^{1/2} t$ for $\zeta < 1$	$\frac{\omega_n(1 - \zeta^2)^{1/2}}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
6. $\frac{d^nf(t)}{dt^n}$	$s^n F(s) - \sum_{i=1}^n s^{n-i} \frac{d^{i-1}f(0)}{dt^{i-1}}$	24. $e^{-\zeta\omega_n t} \sinh \omega_n(\zeta^2 - 1)^{1/2} t$ for $\zeta > 1$	$\frac{\omega_n(\zeta^2 - 1)^{1/2}}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
7. $\int_0^t f(\tau) d\tau$	$\frac{1}{s} F(s)$	25. $1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1 - \zeta^2}} \sin(\omega_n \sqrt{1 - \zeta^2} t + \phi)$	$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$
8. 1 or $u(t)$	$\frac{1}{s}$	$\phi = \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{\zeta}$	
9. t	$\frac{1}{s^2}$	for $\zeta < 1$	
10. t^n for $n > -1$	$\frac{n!}{s^{n+1}}$	$\left. \begin{matrix} f(t-a) & \text{where } t > a \\ 0 & \text{where } t < a \end{matrix} \right\}$	$e^{-as} F(s)$
11. e^{-at}	$\frac{1}{s+a}$		
12. te^{-at}	$\frac{1}{(s+a)^2}$		
13. $t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$		
14. $1 - e^{-at}$	$\frac{a}{s(s+a)}$		
15. $e^{-at} - e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$		
16. $ae^{-at} - be^{-bt}$	$\frac{(a-b)s}{(s+a)(s+b)}$		
17. $\sin at$	$\frac{a}{s^2 + a^2}$		
18. $\cos at$	$\frac{s}{s^2 + a^2}$		



Semi-Logarithmic
4 Cycles x 10 to the Inch



Semi-Logarithmic
4 Cycles x 10 to the Inch