# National Exams – May 2012
# 07-Elec-B10, Electro-Optical Engineering

**3 hours duration**

**NOTES:**

1. If doubt exists as to the proper interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement about any assumptions made.
2. Candidates may use one of two calculators, the Casio or Sharp approved models.
3. This is a **"Closed-Book"** examination. The candidate may have a single 8.5 inch by 11 inch sheet (both sides) of hand-written notes as an aid for the examination.
4. Any **five** questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
5. All questions are of equal value.
6. This examination paper has 4 pages.

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**Values of common constants:**

$\varepsilon_o = 8.854 \times 10^{-12}$ F/m
$\mu_o = 4\pi \times 10^{-7}$ H/m
$c = 2.998 \times 10^{8}$ m/s
$q = 1.602 \times 10^{-19}$ C
$h = 6.626 \times 10^{-34}$ J·s
$K = 1.381 \times 10^{-23}$ J/ °K
0°K = -273°C
1 Å = $1.0 \times 10^{-10}$ m

Si $\varepsilon_r = 11.8$
Si $n = 3.42$
Si $E_g = 1.11$ eV
Ge $\varepsilon_r = 16.0$
Ge $n = 4.01$
Ge $E_g = 0.67$ eV
GaAs $\varepsilon_r = 13.2$
GaAs $n = 3.63$
GaAs $E_g = 1.41$ eV
InGaAsP $n = 3.5$

$LiNbO_3$ $\varepsilon_r = 32$
$LiNbO_3$ $r = 30$pm/V
$LiNbO_3$ $n_o = 2.30$

**Useful formulas:** $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$ $P(n) = \frac{N^n \exp(-N)}{n!}$

$Al_xGa_{1-x}As \quad E_g(eV) = 1.424 + 1.266x + 0.266x^2$

$I_s = R_o\sqrt{P_oP_1}\cos\theta$ $n(E) = n_0 - \frac{1}{2} r n_0^{\,3} E$ $x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

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**Question 1**

(a) A step-index fiber has a core radius $a$ and refractive index $n_1$. The thick cladding has refractive index $n_2 < n_1$. Considering only meridional rays, derive an expression for the intermodal dispersion of the fiber.

(b) A step-index fiber has core diameter 62.5μm, core index $n_1=1.47$ and cladding index $n_2=1.45$. If the wavelength is 850nm, calculate the numerical aperture, the V number, the approximate number of modes which can propagate, and the maximum length of fiber for a data transmission rate of 100 Mb/s with NRZ format.

(c) If the core of the fiber in (b) now has a parabolic index profile with maximum index $n_1=1.47$, calculate the V number, the approximate number of modes which can propagate, and the maximum length of fiber for data transmission rate of 100 Mb/s with NRZ format. Use a wavelength 850 nm.

(d) With the aid of a suitable diagram, explain why each propagating mode in the fiber has an evanescent field in the cladding. What role does this evanescent field play in making fiber couplers?

**Question 2**

(a) Sketch the longitudinal cross-section of an APD, clearly showing the different doped semiconductor regions and the metal electrodes defining the active region. For an applied reverse voltage sufficient for the onset of avalanching, plot the charge distribution and electric field distribution in the device as a function of distance through the device. Clearly label important features of your plots. Explain operation of the device when light is incident upon the surface.

(b) Discuss the factors that affect the responsivity of an APD. Sketch the responsivity of a silicon APD vs. wavelength of incident light, and determine the band edge wavelength.

(c) An APD has a quantum efficiency of 75% at 0.85μm. When illuminated with radiation of this wavelength it produces an output photocurrent of 10μA after avalanche gain with a multiplication factor M=250. Calculate the received optical power to the device. How many photons per second does this correspond to?

(d) Define the noise equivalent power (NEP) of a photodetector. Derive an expression for the NEP of an APD in which the dark current dominates. Calculate the NEP of the device in part (c) when operated under low illumination such that the dark current of 80 nA dominates. The APD has excess noise factor $M^x$ with x=0.3

**Question 3**

(a) Explain the four main steps which lead to spontaneous emission in a laser diode. Use energy level diagrams to illustrate your answer.

(b) Consider a semiconductor Fabry-Perot optical cavity of length 250 μm with end mirrors that have reflectance 85%. The refractive index of the semiconductor is 3.7 .Calculate the cavity mode nearest to the free space wavelength of 1310nm. What is the free-space wavelength of this mode?

(c) For the same cavity, calculate the wavelength separation of the modes in free space, the finesse of the cavity, and the spectral width of each mode in nm in free space.

**Question 3 (continued)**

(d) A 1310 nm wavelength laser diode is to be made using this cavity. The lasing region of the cavity has width 10 μm and thickness 1 μm. The extra cavity losses (not including the mirror losses) are 2 $cm^{-1}$ and the threshold current density is 100 $A/cm^2$. The electron population density at threshold is $2x10^{18}$ $cm^{-3}$. Calculate the photon lifetime, the spontaneous emission lifetime, the threshold current, the threshold gain, the output slope in W/A when the diode is lasing, and the optical power at a current of 10 mA.

**Question 4**

(a) Derive the relationships for the electrical and optical bandwidths of an optical fiber system . (Hint: consider the different ratios of output power at the detector to input power transmitted at the source.) The difference between them in frequency terms depends on the shape of the frequency response. For a Gaussian frequency response, estimate the 3 dB optical bandwidth corresponding to a 3 dB electrical bandwidth of 50MHz.

(b) What is the difference between coherent optical homodyne detection and heterodyne detection? What is OOK modulation? When the local oscillator signal power is much greater then the incoming signal power in a coherent detection scheme, what is the dominant noise in the detector? For simple OOK modulation and using a PIN detector having responsivity *R* derive from first principles the theoretical SNR improvement for optical homodyne detection over heterodyne detection for this case. What is the primary reason for the improvement?

(c) An optical fiber system employs a laser diode transmitter which launches an average optical power of 400μW at a wavelength of 0.85 μm into the optical fiber cable. The cable has an overall attenuation (including splices) of 3.5dB/km. The APD receiver requires 1100 photons in order to register a binary "1" with a BER of $10^{-10}$. Assume there is an equal likelihood of a "1" or "0" being transmitted. Ignoring dispersion, determine the maximum transmission distance (without repeaters) provided by the system when the transmission rate is 1Mb/s, 10 Mb/s, 100 Mb/s and 1Gb/s such that a BER of $10^{-10}$ is maintained. Hence, sketch a graph showing the allowable transmission distance versus the bit rate of the system.

**Question 5**

A *pin* diode operating at wavelength 0.83 μm has an external quantum efficiency of 50% and a dark current of 0.5nA at temperature 295K. The diode is loaded by a transimpedance amplifier having feedback resistor 50 kΩ and open loop gain 32. The diode has resistance 1MΩ and capacitance 1pF. The amplifier has input capacitance 6pF and input resistance 10MΩ.
The desired post-detection bandwidth is 10 MHz.

(a) What is the diode's responsivity at wavelength 0.83 μm?

(b) What is the bandwidth of the diode-transimpedance amplifier combination? Is equalization necessary?

(c) If the required SNR is 55 dB, what incident optical power is required expressed in dBm?

**Question 6**

A digital fiber optic link needs to operate at 550 Mb/s in NRZ format over 45 km of single mode fiber at wavelength 1550 nm without repeaters. A single mode InGaAsP laser diode launches an average optical power -12dBm into the fiber. The laser diode has 1 nm spectral width and the rise time of the transmitter is 0.5 ns. The fiber has a loss 0.33 dB/km and dispersion 2.5 ps/(nm-km). There is a splice with loss of 0.1 dB every kilometer. The coupling loss at the receiver is 0.5 dB and the receiver uses an InGaAs APD with sensitivity of -35dBm. The receiver bandwidth is 500 MHz. Excess noise penalties are predicted to be 1.5 dB.

(a) Set up an optical power budget for this link and find the system margin.
(b) Calculate the system risetime. What can you conclude about the design of the system?
(c) Can the specified system also use RZ format for the data?
(d) What is a WDM optical link? Could the above fiber link be used for WDM? Explain your answer.

**Question 7**

(a) A simple integrated optical phase modulator can be made by fabricating a single buried optic waveguide of width $w_1$ in a electro-optic material between two metal electrodes of length $L$ and width $w_2$ on the surface separated by distance $d$ from each other, and each a distance $g$ from the buried waveguide. A voltage V is applied between the electrodes. Sketch the device and explain its operation as an optical phase modulator.
(b) Compare the voltages required to produce a phase shift of 180 degrees at wavelength 1300 nm when using $LiNbO_3$ or a III-V semiconductor substrate. Use $L$=3 cm, and $d$=30μm. For $LiNbO_3$ use $n_o = 2.30$ and $r = 30$ pm/V. For the III-V semiconductor use $n_o = 3.10$ and $r = 1.3$ pm/V. Comment on the two voltages obtained.
(c) Sketch how you would use this phase modulator to make an integrated optical amplitude modulator. Explain its operation and plot the response of light intensity vs. applied voltage for the device.
(d) What limits the electrical frequency response of the amplitude modulator in part (c)?
(e) What limits the frequency response for direct modulation of a laser diode? Provide approximate values for the frequency limits of direct modulation vs. external modulation.