National Exams December 2012

98-Phys-A7, Optics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate should include in the answer clear statements of the interpretation and any assumptions made.

2. This is a CLOSED BOOK EXAM.

3. Candidates may use one of two calculators, the Casio or Sharp approved models.

4. Answers to question 1 plus any four of questions 2 to 6 constitute a complete exam paper.

5. Answer question 1 in the space provided on the exam paper.

6. The first four questions as they appear in the answer book will be marked.

7. Each question is of equal value. Question 1 is mandatory.

1. A phrase, or a diagram and a phrase, is all that is required in most cases. [20 marks total, one mark for each letter]

a) This examination is about light and optics. Define *light*.

b) Define *optics*.

c) Define *geometrical optics*.

d) Define *physical optics*.

e) What problems with *geometrical optics* led to the development of *physical optics*?

f) Define *plane of incidence*.

g) State the *law of reflection*.

h) State the *law of refraction.*

i). Complete any two rows of the following table:

		wavelength range (nm)	frequency range (Hz)
	UV light		
	red light		
	blue light		
	IR light		

j) Transverse plane waves are solutions to Maxwell's equations for linear, homogeneous, isotropic, stationary media with $\rho_f = 0$ and $J_f = 0$. Define *transverse plane wave.*

k) Is analysis in terms of monochromatic plane waves restrictive?

l) How is the ray of geometrical optics related to the plane wave of physical optics?

m) What is the speed of propagation of the wave $f(x, t) = A\exp(\alpha x)\sin(2\pi(at + bx))$?

n) Write an equation for the *Poynting vector* and give the SI units for the quantity the Poynting vector represents.

o) Define *numerical aperture*.

p) Define *f-number*.

q) What is the difference between *interference* and *diffraction*?

r) Define *evanescent wave*.

s) What is *Fraunhoffer* diffraction?

t) What is *Fresnel* diffraction?

2. [20 marks total]

a) Write equations for the electric field and magnetic fields for TE and TM polarized plane waves. Let the plane of incidence be the x-y plane and allow the waves to be propagating at an angle of θ with respect to the y axis. Remember to draw the coordinate system. [5 marks]

b) Check your answers for the **E** and **H** fields and **k** for part a) by taking limiting cases and by calculating **E**×**H**. Explain your reasoning. [4 marks]

c) Find the irradiance in the $y = 0$ plane given two TE polarized plane waves of equal amplitude. Let the plane of incidence be the x-y plane and assume that one plane wave is travelling at an angle of θ with respect to the y axis and the other plane wave is travelling at an angle of $-\theta$ with respect to the y axis. Assume (i) that the waves interfere coherently and (ii) incoherently. How does the answer change if both plane waves were TM polarized? [11 marks]

3. [20 marks total]

a) Design a 5 × beam expander spatial filter for a HeNe laser with a wavelength of 632.8 nm. Assume that the input beam is nominally a plane wave and that the input aperture is 5 mm in diameter. The output beam should be a plane wave. Draw an engineering diagram of your design. Specify all relevant dimensions and parts, and justify with calculations the values for the dimensions. Assume that the glass for the lens is BK7 and has a refractive index of 1.50. **Hint:** A beam expander spatial filter is a telescope composed of two lenses with an aperture in the common focal plane that passes the on-axis plane wave and rejects other plane waves. [8 marks]

b) Write the total transfer matrix for the optical system of part a) but leave it as a product of matrices. The total transfer matrix M_T for one design is given below where p is the distance to the first optical element, f is a focal length, q_2 is the distance from the last optical element, and a is a constant. ***Deduce*** (i.e., use equations or ray diagrams to show that it must be so) what information each element in M_T gives and thus ***deduce*** the value of the constant a. [8 marks]

$$M_T = \begin{bmatrix} -a & \dfrac{-a^2 p + a^2 f + a f - q_2}{a} \\ 0 & \dfrac{-1}{a} \end{bmatrix}$$

c) Use ray tracing to show how spherical aberration and chromatic aberration would affect your design. [4 marks]

4. [20 marks total]

a) List the possible unique polarizations of light and describe the effect of a linear polarizer and quarter wave plate on each of the unique polarizations that you list. [6 marks]

b) Deduce the nature of light that is consistent with analysis. When a polarizer is rotated in the path of the light, there is some intensity variation but no position of the polarizer that gives zero intensity. The polarizer is set to give maximum intensity. A quarter wave plate is placed in front of the polarizer. Rotation of the quarter wave plate can now produced zero intensity. [4 marks]

c) Assume air and water with a refractive index of 1.33 Sketch carefully *R* and *T* for both polarizations as a function of the angle of incidence for internal reflection. Label the salient features and calculate numerical values for the salient features on both the vertical and horizontal axes. [4 marks]

d) Assume air and water with a refractive index of 1.33 Sketch carefully *R* and *T* for both polarizations as a function of the angle of incidence for external reflection. Label the salient features and calculate numerical values for the salient features on both the vertical and horizontal axes. [4 marks]

e) Use your results to explain the conditions wherein polarizing sun glasses are most effective. State the orientation of the transmission axis of the sun glasses under normal conditions of use. [2 marks]

5. [20 marks total]

a) Define coherence, state the principle of linear superposition, and state Rayleigh's criterion for resolution. [5 marks].

b) Using explicitly the concepts introduced in a), find the distance from an observer that two headlights on a car will be just resolvable. Assume a wavelength of 550 nm, a pupil diameter of 5 mm, and a focal length of the eye of 15 mm. Draw a ray diagram of the optical system and clearly label the diagram. [6 marks]

c) Explain, using a sketch and the Huygens-Fresnel Principle, how the intensity distribution in the focal plane of a lens is calculated. Assume a circular lens with *f*/# = 5, focal length of 10 cm, and illumination from a point source 20 cm in front of the lens. Give the dimensions of salient features in the focal plane. [9 marks]

6. [20 marks total]

A plot of the base 10 logarithm of the normalized far field for illumination of a transmission grating at normal incidence with monochromatic light is given below. The plot is given as a function of the far field angle θ, with θ measured in radians.

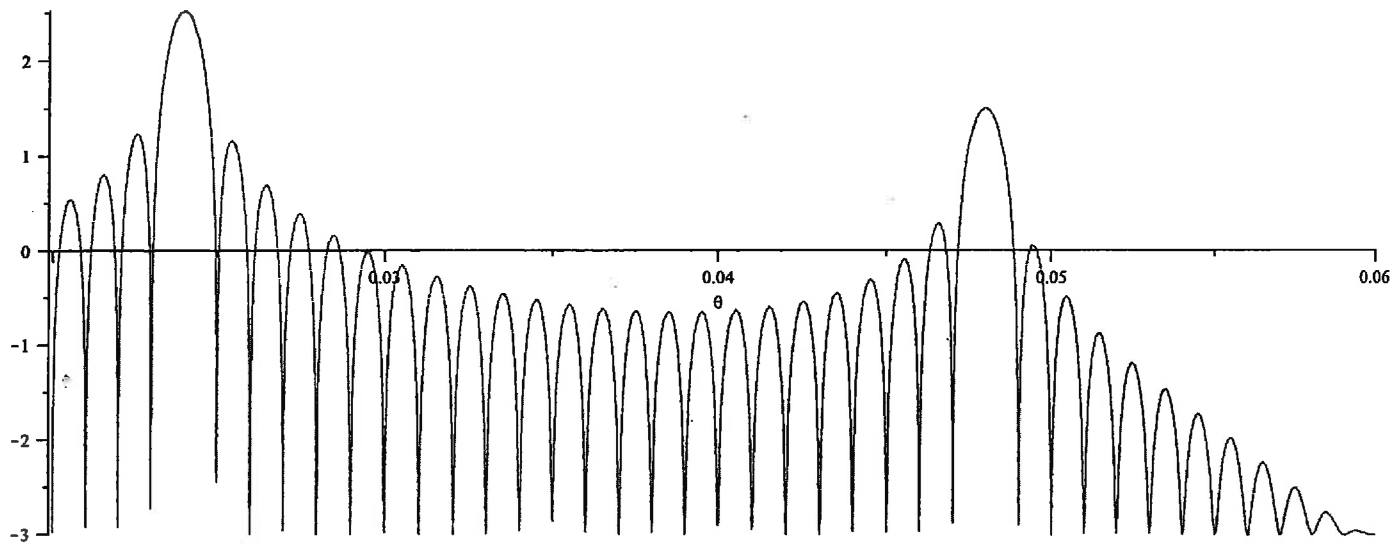
a) Determine the number of lines in the grating, the width of an individual slit, and the spacing of the slits in the grating. Assume a wavelength of 600 nm. [10 marks]

b) Two parallel slits, which are separated by 0.5 mm, illuminated by a monochromatic plane wave of wavelength 600 nm. [10 marks]

(i) If it is desired to have a fringe spacing of 1.0 mm at a screen, how far behind the slits must the screen be placed?

(ii) If a thin plate of glass with n = 1.50 is placed in front of one of the slits, what is the lateral fringe displacement at the screen?

Question 6 b) is the last question. Some formulae follow after the figure.



$$E(x,y,z) = \frac{ik}{2\pi}\frac{e^{ikz}}{z} e^{i\frac{k}{2z}(x^2+y^2)} \iint E(x_a,y_a,0)\, e^{i\frac{k}{2z}(x_a^2+y_a^2)}\, e^{-i\frac{k}{z}(x\,x_a+y\,y_a)}\, \mathrm{d}x_a\, \mathrm{d}y_a$$

The field in the neighbourhood of the focus of a circular lens of radius a is given in the usual paraxial approximation as

$$E(u,v) = \int_0^1 \mathrm{J}_0(2\pi v \rho_a)\, e^{-i\pi u \rho_a^2}\, \rho_a\, \mathrm{d}\rho_a$$

with $\rho_a = \sqrt{(x_a^2 + y_a^2)}$, $u = \frac{1}{\lambda}\frac{a^2}{q\,(q+\Delta)}\Delta$, $v = \frac{1}{\lambda}\frac{a\,\sqrt{(x^2+y^2)}}{(q+\Delta)}$.

$J_0(0) = 1$; $J_0(2.4048) = 0$; $J_0(5.5201) = 0$; $J_0(8.6537) = 0$; $J_0(11.7915) = 0$;

$\gamma = \frac{1}{2}\, k\, D \sin(\theta)$. The zeros for $J_1(\gamma)$ occur for γ = 0, 3.832, 5.136, 7.016, 8.417, 10.173, 11.620, 13.324, . . .

$$\frac{d}{dx}\, x^n\, J_n(x) = x^n\, J_{n-1}(x)$$

$$(1+\epsilon)^{\xi} = 1 + \frac{\xi}{1!}\epsilon + \frac{\xi\times(\xi-1)}{2!}\epsilon^2 + \ldots$$

zone plate radii $R_m = (m\, r_o\, \lambda)^{0.5}$

The intensity in the far-field as a function of the angle θ_m from the normal of a diffraction grating of N lines, line spacing of a, and line width of b, for illumination with a plane wave with $k = 2\pi/\lambda$ and an angle of incidence of θ_i is

$$I(\theta) = I_o\left[\frac{\sin(\beta)}{\beta}\right]^2\left[\frac{\sin(N\alpha)}{\sin(\alpha)}\right]^2$$
$$\beta = \frac{kb}{2}(\sin(\theta_i) + \sin(\theta_m))$$
$$\alpha = \frac{ka}{2}(\sin(\theta_i) + \sin(\theta_m))$$

For a blazed grating, $2\theta_b = \theta_i - \theta_m$

The resolution R and the dispersion D for a grating with N lines and order m are

$$R = \frac{\lambda}{\Delta\lambda} = mN \qquad D = \frac{m}{a\cos(\theta)}.$$

double angle formulae:

$$\sin(A+B) = \sin(A)\sin(B) + \cos(A)\cos(B)$$
$$\sin(A-B) = \sin(A)\sin(B) - \cos(A)\cos(B)$$
$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin(A) - \sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos(A) - \cos(B) = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$v_p = \frac{\omega}{k} \qquad v_g = \frac{d\omega}{dk}$$

The translation, refraction at a spherical interface, thin lens, and spherical mirror matrices are listed below.

$$\begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ \frac{n_1 - n_2}{Rn_2} & \frac{n_1}{n_2} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ \frac{-2}{R} & 1 \end{bmatrix}$$

THE END