

National Exams December 2012

98-Ind-A1

Operations Research

3 hours duration

NOTES:

- 1 If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
- 2 Any non-communicating calculator is permitted. This is an Open Book exam. Note to candidates: You must indicate the type of calculator being used. i.e. write the name and model designation of the calculator, on the first left hand sheet of the exam workbook.
- 3 There may be more questions than you are able to answer in the allotted time. Although the total value of the questions is 200, any marks achieved will be considered toward the 100 total requirements.

1. Consider the LP: $\text{Max } z = 5x_1 + x_2 + 2x_3$ such that $x_1 + x_2 + x_3 \leq 6$; $6x_1 + x_3 \leq 8$; $x_2 + x_3 \leq 2$; $x_1, x_2, x_3 \geq 0$

After adding the slack variables x_4, x_5 and x_6 , and solving by substitution we obtain the following final equations:

$$z = 9 - (1/6)x_2 - (5/6)x_5 - (7/6)x_6$$

$$x_1 = 1 + (1/6)x_2 - (1/6)x_5 + (5/6)x_6$$

$$x_3 = 2 - x_2 - x_6$$

$$x_4 = 3 - (1/6)x_2 + (1/6)x_5 + (5/6)x_6$$

- 10 a. Find the dual, and the optimal solution of the dual, to this LP.
- 5 b. Find the range of values of the cost coefficient of x_1 for which the basis remains optimal
- 5 c. Find the range of values of the cost coefficient of x_2 for which the basis remains optimal
- 20 2 We are thinking of filming the Don Harnett story. We know that if the film is a flop we will lose 4 M\$, and if the film is a success we will earn 15M\$. We currently believe there is a 10% chance that the film will be a success. Before filming we have the option of paying the noted movie critic Roger Alert 1 M\$ for his view of the film. In the past, Alert has predicted 60% of all actual hits to be hits, and 90% of all actual flops to be flops. Assuming that we want to maximize our expected profit, should we pay Alert his exorbitant fee?
- 3 A company has one item in its inventory that requires special storage. The company estimates the storage cost at \$2.00 per unit per year. The demand for the item is 40,000 units per year, and the ordering cost is \$16.00 per order.
- 10 a. Assuming shortages are not allowed, write an expression for the total yearly cost in terms of the order quantity Q , and determine the optimal order quantity.
- 10 b. Assuming shortages are allowed and cost \$4.00 per unit per year, write an expression for the total yearly cost in terms of the order quantity Q and the maximum shortage level s , and determine the optimal order quantity.

- 20 4 A company that conducts surveys, is going to bid on a job that has the following requirements
- There must be at least 300 personal interviews
 - There must be at least 500 interviews (personal or telephone) conducted at night.
 - Of the day interviews, at least 60% of them must be by telephone.
 - There must be at least 1000 total interviews ((personal or telephone)

The cost of doing an interview is as follows:

	Personal	Telephone
Day	\$2.00	\$1.00
Night	\$2.10	\$1.20

In preparing the bid the company wants to know the minimum cost of meeting the requirements. Define the decision variables, and set up the linear programming model. Do not solve.

- 20 5 The sales manager of a publisher of university textbooks has six travelling sales staff to assign to three different regions. She has decided that each region should be assigned one or more dedicated sales staff. The estimated sales per region varies with the number of staff assigned as follows:

No. of sales staff	Region 1	Region 2	Region 3
1	35	21	28
2	48	42	41
3	60	56	53
4	69	70	65

Use dynamic programming to determine how the six travelling sales staff should be assigned to the three regions to maximize the total sales.

- 6 A single overhead crane attends to re-load ten machines. When a machine finishes its load, the overhead crane is called to unload the machine and to provide it with a new load from an adjacent storage area. The machine time per load is assumed exponential with mean 30 minutes. The time from the moment the crane moves to service a machine until a new load is installed is also exponential with mean 10 minutes.
- 10 a. What percentage of time is the cradle idle?
- 10 b. What is the expected number of machines waiting for crane service?

7 An oil company is installing an oil pipeline from an oil field to a refinery. The pipeline requires the welding of 1000 seams, to be carried out by the company's own welders. Defective seams result in leaks, which must be reworked at a cost of \$1,200 per seam. It is estimated from past experience that 5% of the seams will be defective with probability 0.30, or 10% will be defective with probability 0.50, or 20% will be defective with probability 0.20. The company can also hire an expert cleanup team of welders at a one-time cost of \$130,000, who would check all of the welds done by the company welders and repair them as required.

- 10 a. Based on an expected value criterion, should the company bring in the expert clean-up team to check and rework the welds, or repair the welds as they occur?
- 10 b. The company can also improve its information about the quality of its own welders on this job, by x-ray inspection of a randomly selected completed weld at a cost of \$2,000. Is it worthwhile to carry out this inspection?

8 Every time a machine breaks down it requires 1, 2, or 3 hours to fix it according to the following probability distribution.

Repair time (hours)	Probability
1	0.30
2	0.50
3	0.20

The number of machine breakdowns per day is given by

Machine breakdowns per day	Probability
0	0.30
1	0.30
2	0.40

- 10 a. Draw a flowchart for a procedure that will estimate the average amount of work required in a day to fix machines.
- 10 b. Using the following random number list, carry out the simulation for two days operation, and compare the simulated average with the theoretical average amount of repair work per day.

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- 9 A particular project consists of nine tasks. The crash times, normal times, b_{ij} values giving the cost increase associated with a unit time saving for activity (i,j), and precedence relationships are given below:

Task	Min Task Time	Normal Task Time	b_{ij}	Immediate Predecessors
A	1	3	4	none
B	2	4	1	none
C	0.5	2	1	A
D	2	5	1	A
E	1	6	3	B, C
F	1	2	7	D, E
G	3	4	9	D, E
H	2	3	5	F
I	4	5	8	G

- 10 a. Draw an appropriate project diagram, and identify the tasks on the critical path.
- 10 b. Write, but do not solve, an LP model for the problem of determining optimal activity times for completing the project by a specified deadline.
- 20 10. Consider an electricity company with a 100 MW generating unit. The unit has a long history of failures which often restrict its output to 75, 50, 25 or 0 MW. After examining the historical records you somehow determine that the probability of failures is only a function of the current state, and does not depend in any way on what happened before. Thus you believe a Markov Chain model is appropriate. Whenever it is producing any output there is a 5% probability that in the next hour it will totally fail and be forced out of service. After one hour of repair there is a 50% chance that it will still be unable to produce any electricity in the following hour, a 40% chance it will return to produce 100 MW in the next hour, a 6% chance it will produce 75 MW, a 3% chance it will produce 50 MW and a 1% chance it will produce only 25 MW. What is the long-term expected output of this unit?