National Exams May 2011

07-Mec-B6, Advanced Fluid Mechanics

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any non-communicating calculator. The exam is OPEN BOOK.
3. Answer any 3 of the 4 questions in Part **A** and any 2 of the 3 questions in Part **B**
4. Weighting: Each question is equally weighted within a section.
   Part **A**: 40%; Part **B** 60%

Part A: Answer any **3** of the following 4 questions.

Question A1: A massless L-shaped gate is hinged at **A** as shown in Fig. A1. Determine the moment about the hinge as a function of the gate horizontal length a and water level h. Is there a critical water level at which the gate will open? If so, what is the critical height? The gate has a uniform width of b (into the page). For water, $\rho = 1000$ kg/m^3.

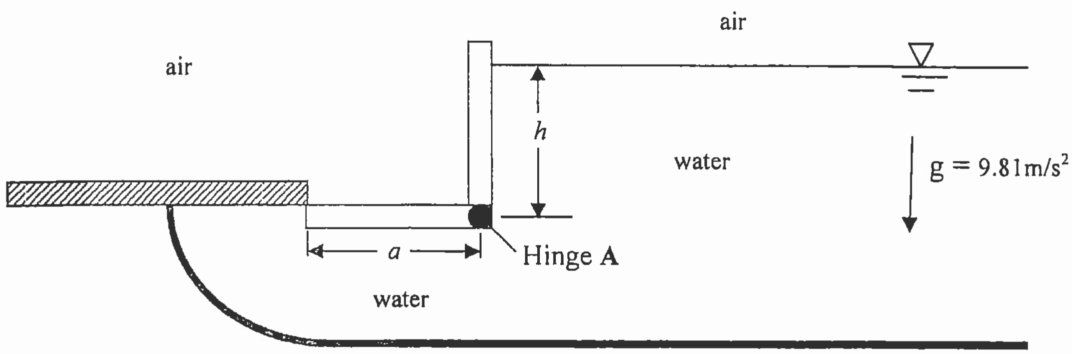


Figure A1: Schematic of L-shaped weir.

Question A2: An explosion creates a normal shock wave propagating in still air at 27°C and 100kPa. The gauge pressure after the passage of the shock is measured to be 1300kPa (gauge). Determine the speed of propagation of the shock, the temperature and speed of the air after the shock. For air: $R = 287$ J/kg-K; $\gamma = 1.4$.

Question A3: A company manufactures geometrically similar airplane propellers up to 4.0m in diameter. Wind tunnel tests for a 0.33m diameter model propeller are performed at an air velocity of 50m/s. For a model rotational speed of 2000rpm, the measured thrust is 100N and the propeller input torque is 20N-m. Calculate the corresponding rotational speed, thrust and input torque of a 4.0m production propeller at an air speed of 150m/s. Reynolds and Mach number effects can be assumed negligible. The air density is the same for both the model and the production propeller.

Question A4: Water flows through a transition section of a canal as shown in Fig. A4. Upstream of the transition, the channel is 5m wide and the water is 4m deep. The water velocity is 1m/s. Far downstream of the transition, the channel is 2m wide and the water flows at a rate of 2.9m/s. Find the force (streamwise component) acting on the walls of the transition. The velocity distribution can be assumed uniform far upstream and downstream of the transition. For water, $\rho = 1000\ kg/m^3$.

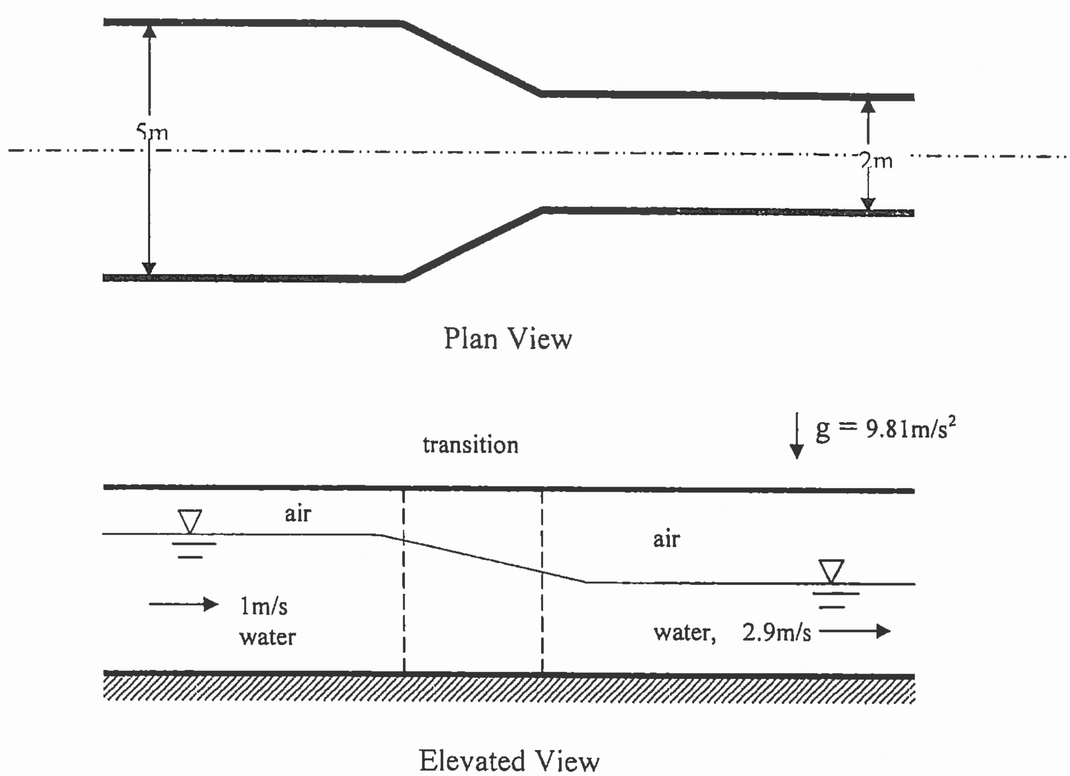


Figure A4: Schematic of canal with transition section.

<u>Part B:</u> Answer any <u>2</u> of the following 3 questions.

Question B1: Water flows over a flat plate surface at a constant rate. A pump draws off water (ρ = 1000 kg/m^3) through a narrow slit at a rate of 0.1 m^3/s per meter length of the slit. Assuming that the fluid is treated as incompressible and irrotational (i.e. potential flow); that the flow field can be approximated as two-dimensional, then the resulting flow field can be approximated by the combination of a uniform flow, at a speed of U_o determined far upstream of the slit and a sink. Hence, the flow field can be represented by the streamfunction:

$$\psi = U_o y - m \tan^{-1}\left(\frac{y}{x}\right) = U_o r \sin\theta - m\theta \quad .$$

a) Determine an expression for the local velocity components in Cartesian, u, v, and cylindrical polar, u_r, u_θ, components.
b) What is the strength of the source m?
c) Given that an open U-tube manometer, filled with mercury (SG=13.6), has ends located at the stagnation point, **A**, and at **B** (0.5m downstream of the slit), what is the speed of the flow directly over the manometer at **B** if the height difference in the mercury column is 2.4cm? From this result, what is U_o ?
d) Where is the stagnation point located relative to the slit? Sketch the flow. What is the value of the streamline corresponding to the stagnation streamline?
e) What is the maximum height, **H**, at which fluid will get sucked into the slit?

Neglect any gravitational effects (except for manometer, of course).

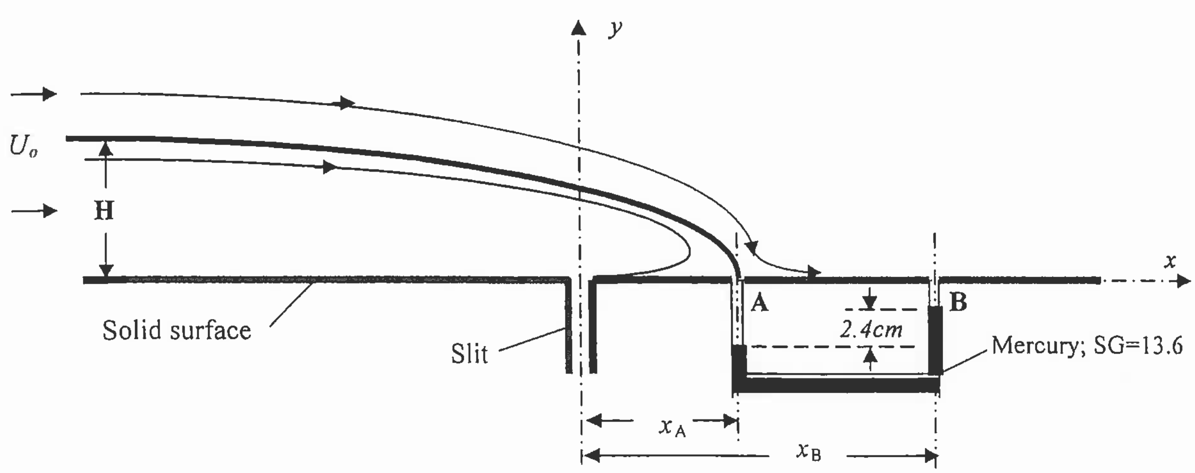


Figure B1: Water flowing over a flat plate with sink at x = 0.

Question B2: An incompressible fluid flows past a horizontal impermeable flat plate (i.e. no flow through the plate) as shown in the figure below. The flow is two-dimensional (i.e. no velocity component into/out of the page), the inlet profile is constant, $u = U_o$, and the exit profile has the form of a cubic polynomial:

$$u = U_o\left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right) \qquad \text{where : } \eta = \frac{y}{\delta} .$$

a) Compute the volumetric flow rate across the top of the control volume spanning $y = 0$ and $y = \delta$. The plate has a width of b into the page.
b) Estimate the drag (friction force) on the top face the plate.

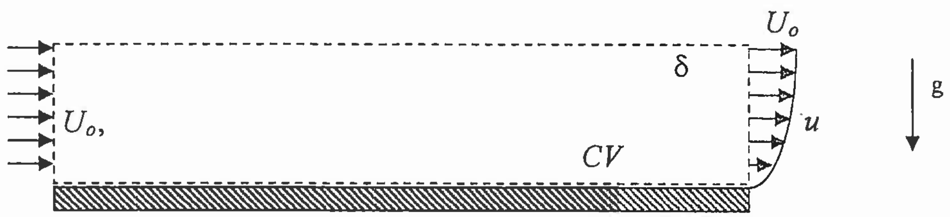


Figure B2: Boundary layer flow over a flat plate.

Question B3: At what height, H, above the lake surface level must the water level in the tower be located to maintain a total flow rate of $10 m^3/s$. What is the flow rate in each of the branches? The pipes are all of inner diameter 100cm and consist of commercial steel of equivalent roughness 0.045mm. The pressure above the surface in the reservoir is ambient. Neglect major losses downstream of the branch (Tee junction). The inlet to the network may be assumed to be ideal. For water use a dynamic viscosity of 0.001 Pa-s and density of 1000 kg/m^3.

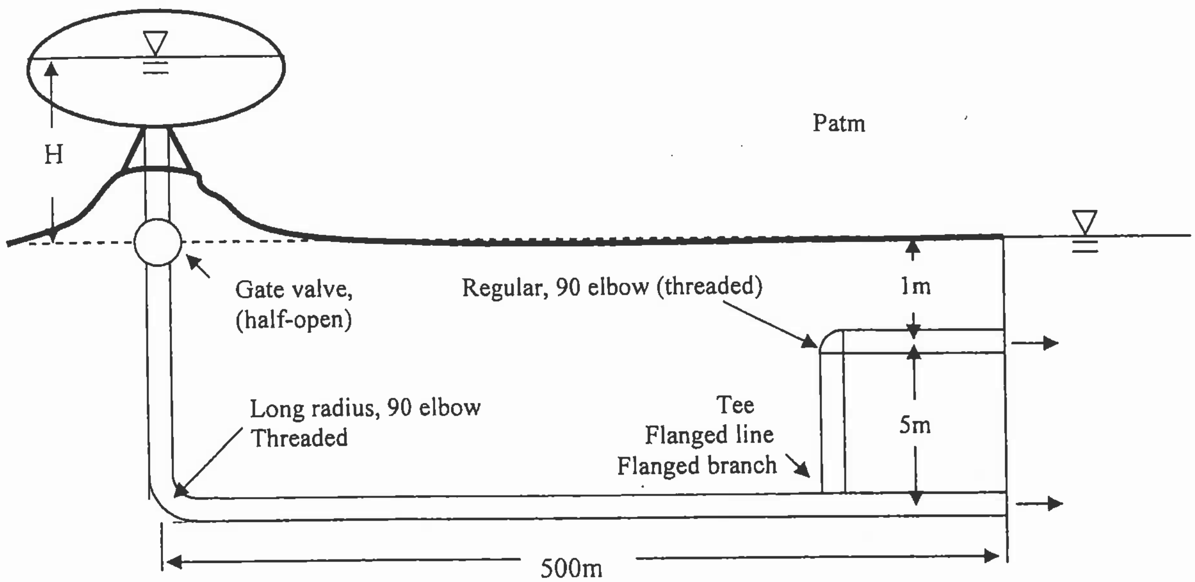


Figure B3: Schematic of pipe network from tower to lake.

MAJOR LOSSES

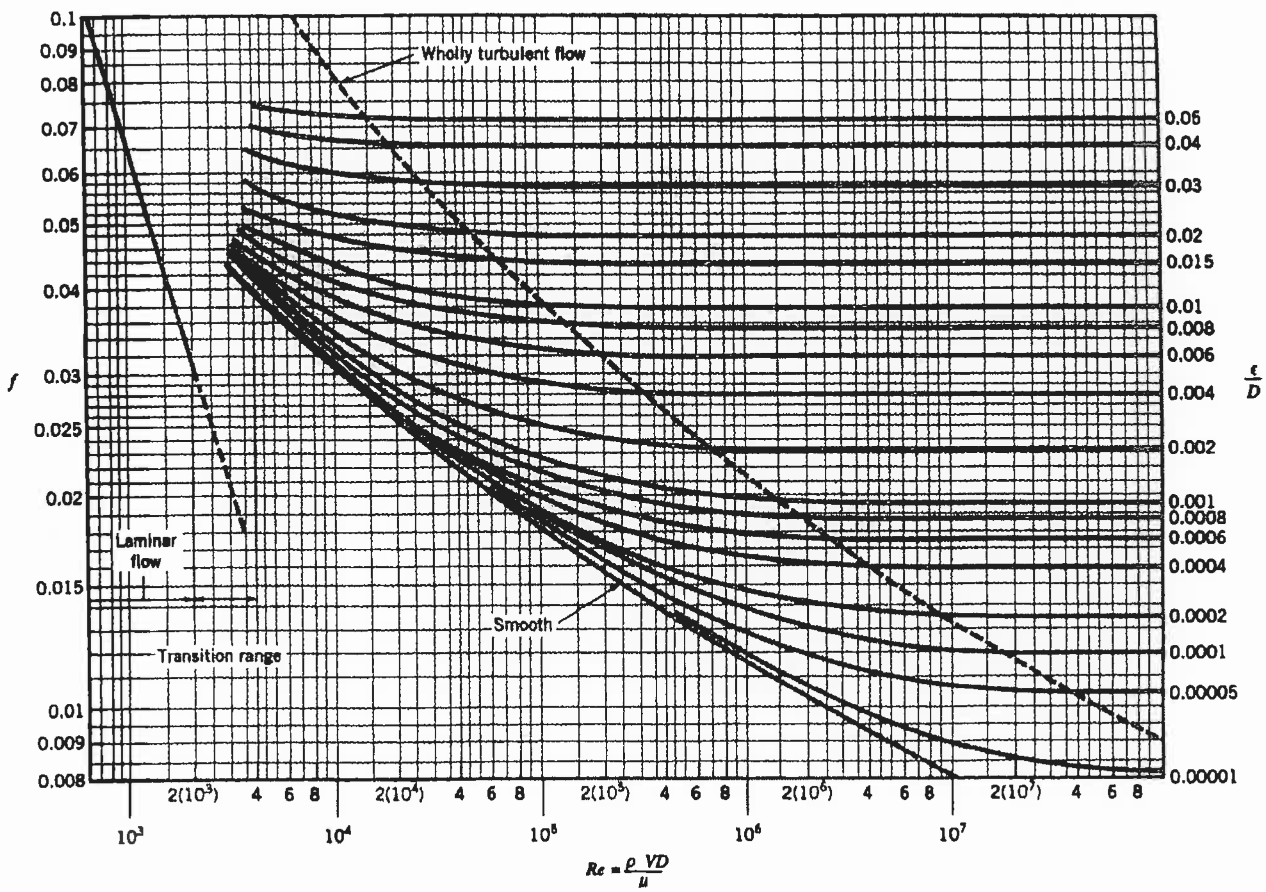
FRICTION COEFFICIENTS:

Colebrook Equation: $$\frac{1}{\sqrt{f}} = -2.0\log_{10}\left[\frac{\varepsilon / D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right]$$

Laminar Flow Equation: $$f = \frac{64}{\mathrm{Re}} \qquad \mathrm{Re} < 2000$$

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

MOODY CHART for PIPE FRICTION



MINOR LOSSES

Loss Coefficients for Pipe Components $\left(h_L = K_L \frac{V^2}{2g}\right)$

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
***e. Valves**	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210