Professional Engineers of Ontario

Annual Examinations – May 2011

07-Elec-B3
Digital Communication Systems

3 Hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.
2. This is a closed book exam. A Casio of Sharp approved calculator is permitted.
3. There are **5 questions** on this exam. **Any 4 questions constitute a complete paper.** Only the first 4 questions as they appear in your answer book will be marked.
4. Marks allocated to each question are noted in the left margin. A complete paper is worth 100 marks.

(25 marks) 1. This question concerns source coding.

(5 marks) a. Suppose you have a source with five letters: A, B, C, D, E, and F; with probabilities given by Pr(A)=0.1, Pr(B)=0.2, Pr(C)=0.4, Pr(D)=0.11, Pr(E)=0.15, Pr(F)=0.04. Calculate the entropy of this source.

(12 marks) b. Obtain a Huffman code for the source in part a, and calculate its average length.

(4 marks) c. Consider the following code for the source in part a: A=11, B=0, C=1, D=10, E=01, F=00. Is this code a good alternative to the code from part b? Briefly explain.

(4 marks) d. Briefly explain one way to find a Huffman code that improves on the code you found in part b.

(25 marks) 2. This question concerns error-control coding.

(5 marks) a. Consider a binary code with the following generator matrix:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Find the corresponding parity check matrix.

(10 marks) b. Find the minimum Hamming distance of this code.

(2 marks) c. Given your answer from part b, how many errors can this code correct? How many errors can the code detect?

(8 marks) d. Using an example, illustrate how the code can correct a single bit error.

(25 marks) 3. This question considers link budgeting.

(10 marks) a. Consider a wireless system with transmitter power of 1 W, antenna gains of 3 dB, receiver losses of 6 dB, receiver noise figure of -168 dBm/Hz, a bandwidth of 1 MHz, and a fading margin requirement of 9 dB. Aside from free-space losses, no other gains or losses are present other than path loss. If the receiver requires a signal-to-noise ratio of at least 6 dB, what is the maximum allowed path loss (in dB)?

(10 marks) b. Using a free-space path loss of $20 \log_{10}(4\pi df/c)$, where d represents the distance from transmitter to receiver, f represents the carrier frequency, and c represents the speed of light ($c = 3.0\times10^{8}$ m/s), and assuming a carrier frequency of 1 GHz, find the maximum distance between transmitter and receiver given the system in part a.

(5 marks) c. Discuss the role of the path loss exponent in modifying the free-space path loss in part b.

(25 marks) 4. This question concerns sampling and D/A conversion.

(5 marks) a. On an audio CD, the audio signal is sampled at a rate of 44.1 kHz. Using the Nyquist sampling criterion, what frequency range of signals can be exactly represented by this sampling frequency?

(10 marks) b. Briefly explain pulse code modulation (PCM). If PCM is used to encode an audio CD signal with 16 bits per sample, what is the required data rate to represent the signal?

(5 marks) c. Suppose PCM is used to sample a signal restricted between -5 V and +5 V. The maximum allowed quantization error is 0.01 V. How many bits per sample are required?

(5 marks) d. Briefly explain how delta modulation is used in analog-to-digital and digital-to-analog conversion.

(25 marks) 5. This question concerns signal modulation and detection.

(4 marks) a. Consider signals $s_0(t)$ and $s_1(t)$, which are used to modulate the binary symbols "0" and "1", respectively, where

$$s_0(t) = \begin{cases} 1, & 0 \leq t \leq T, \\ 0, & t < 0, t > T, \end{cases}$$

and $s_1(t) = -s_0(t)$. Sketch the two signals, and sketch the impulse response of the matched filter $m(t)$ (assuming the filter is matched to $s_0(t)$, and assuming the filter output is sampled at time T).

(8 marks) b. Sketch the convolution of $s_0(t)$ and $m(t)$ as a function of t.

(8 marks) c. Assume these signals are observed through an additive white Gaussian noise process $N(t)$ with power spectral density $S_N(f) = N_0/2$ (i.e., constant with respect to f). Let $Z(t)$ be the output of the matched filter, where the input is the signal plus noise. Give the mean and variance of the random variable $Z(T)$ (i.e., at the sampling instant T), given that 0 was sent, and given that 1 was sent.

(5 marks) d. Give the optimal decision rule for the system assuming that 0 and 1 are equiprobable, and describe how to calculate the probability of error (but don't do the calculation).