National Exams 2011

07-Elec-A3, Signals and Communications

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is a Closed Book Exam but one aid sheet is allowed written on both sides. An approved calculator is permitted.
3. Five (5) questions constitute a complete paper. The first five questions as they appear in the answer book will be marked.
4. All questions are of equal value.
5. Clarity and organization of the answer are important.

1. (20 marks) *This problem has three independent parts*

   (a) (5 marks) Fourier Transform of signal $z(t)$ is

   $$Z(jw) = \frac{e^{-3j\omega}}{j\omega} + \frac{e^{-3j\omega} - 1}{\omega^2}$$

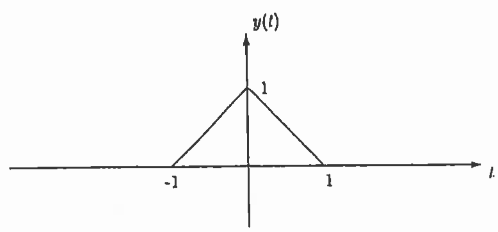
   What are real part and imaginary part of $Z(j\frac{\pi}{4})$?

   (b) (7 marks) Consider the following signal

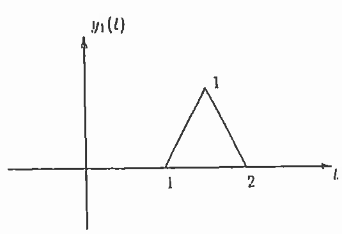
   $$x(t) = 4\delta(t) + \sin(0.3\pi t)\,(u(t) - u(t-1))$$

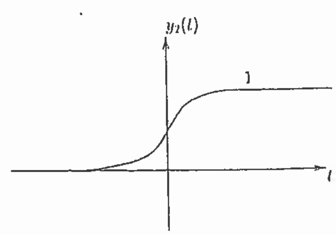
   Find $X(j\omega)$ the Fourier Transform of $x(t)$.

   (c) (8 marks) Consider the following signal $y(t)$: Assume that the Fourier Trans-



form of y(t) is $Y(j\omega)$. Express the Fourier Transforms of $y_1(t)$ and $y_2(t)$ in terms of $Y(j\omega)$.

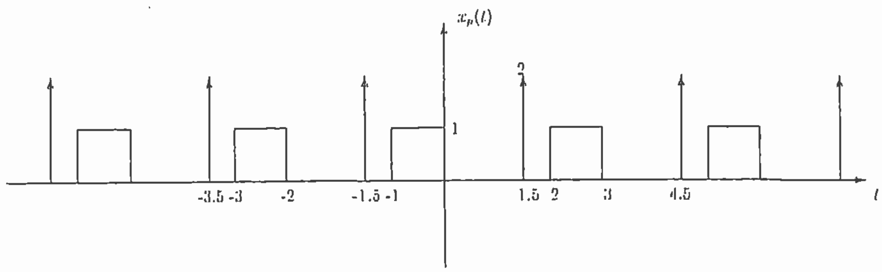




where the closed form expression of $y_2(t)$ is

$$y_2(t) = (0.5t^2+t+0.5)(u(t+1)-u(t))+(-0.5t^2+t+0.5)(u(t)-u(t-1))+u(t-1)$$

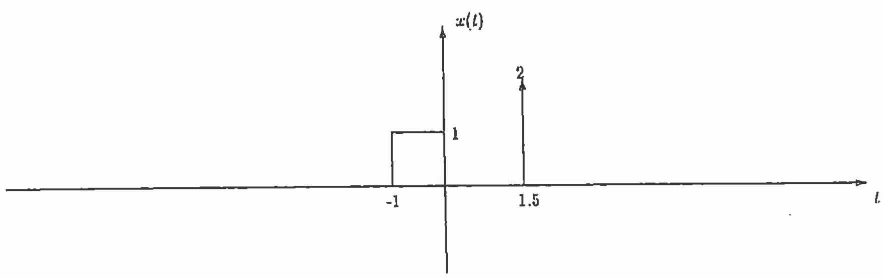
2. (20 marks) Consider the following periodic signal $x_p(t)$



(a) (2 marks) What is the fundamental period of this signal?

(b) (12 marks) Find the Fourier series coefficients of this signal, D_n, and plot the magnitude and phase of D_n for $-10 \leq n \leq 10$.

(c) (6 marks)(Independent from previous parts).
What is the relationship between the Fourier Transform of signal $x(t)$ and the Fourier series coefficients of $x_p(t)$, D_n.

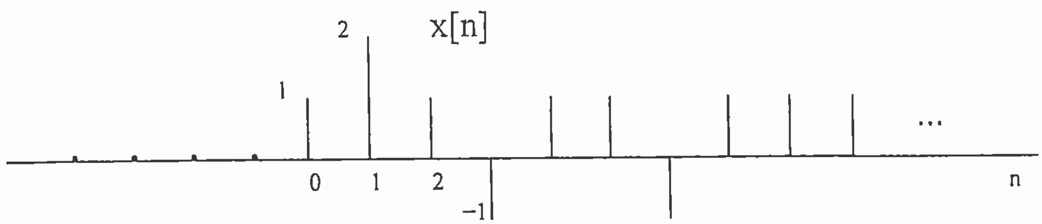


3. (**20 marks total**) Consider a linear time-invariant system whose system transfer function $H(z)$ is

$$H(z) = \frac{z^{-2}}{(1 - \frac{1}{2}z^{-1})(1 - 3z^{-1})}$$

(a) (**5 marks**) Suppose the system is known to be stable. Determine the output $y[n]$ when the input $x[n]$ is the unit step sequence.

(b) (**7 marks**) Suppose the region of convergence of $H(z)$ includes $z = \infty$. Determine $y[n]$ evaluated at $n = 2$ when $x[n]$ is as shown in the below Figure.



(c) (**8 marks**) Suppose we wish to recover $x[n]$ from $y[n]$ by processing $y[n]$ with an LTI system whose impulse response is given by $h_i[n]$. Determine $h_i[n]$. Does $h_i[n]$ depend on the region of convergence of $H(z)$?, explain.

4. (Total 20 marks) Consider the modulated signal

$$x_M(t) = 500\cos(10000\pi t)[1 + \sum_k \frac{1}{2k}\cos(1000k\pi t)], k = 1, 2, 5$$

(a) (4 marks) What frequencies are present in the modulated signal?

(b) (4 marks) What are the magnitudes of the components at each frequency?

(c) (4 marks) What is the total transmitted power?

(d) (4 marks) What is the modulation index?

(e) (4 marks) What is the efficiency?

5. (Total 20 marks) The following signals are used to generate DSBSC (double-sideband suppressed carrier) AM signals. Which signals can be recovered using envelope detection?

   (a) (5 marks) $x(t) = \cos(2\pi f_1 t)$

   (b) (5 marks) $x(t) = 2\cos(2\pi f_1 t) + \cos(2\pi f_2 t)$

   (c) (5 marks) $x(t) = 3 + \cos(2\pi f_1 t)$

   (d) (5 marks) $x(t) = 3 + 2\cos(2\pi f_1 t) + \cos(2\pi f_2 t)$

6. (Total 20 marks) A modulated RF waveform is given by $500\cos[w_c t + 20\cos w_1 t]$, where $w_1 = 2\pi f_1$, $f_1 = 1$ kHz, and $f_c = 100$ MHz.

   (a) (10 marks) If the phase deviation constant is 100 rad/V, find the mathematical expression for the corresponding phase modulation voltage $m(t)$. What is its peak value and its frequency?

   (b) (10 marks) If the frequency deviation constant is 1×10^6 rad/V-s, find the mathematical expression for the corresponding FM voltage $m(t)$. What is its peak value and its frequency?