National Exams December 2011

04-BS-6: Mechanics of Materials

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.

2. Candidates may use one of two calculators, the Casio or Sharp approved models.

   This is a Closed Book exam. However candidates are permitted to bring the following into the examination room:

   - ONE aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae. **Example problems and solutions to problems are not allowed!**

3. Any five questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

NOTE: The aid sheet must be handed in with the exam!

Your exam will not be marked if you do not hand in an aid sheet, unless there is a signed statement by the exam invigilator stating that no aid sheet was used for the exam.

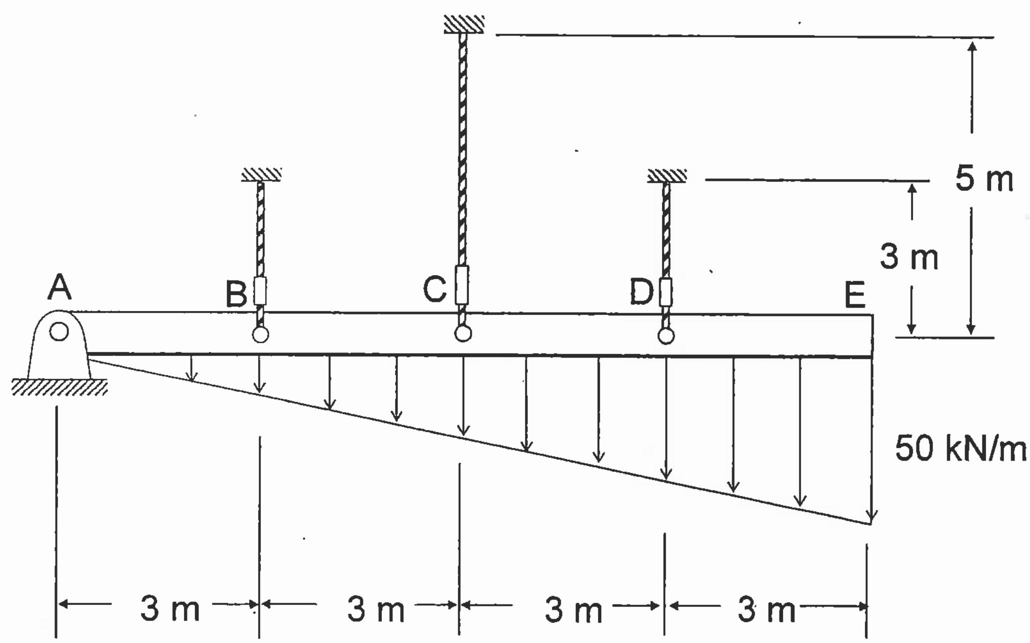
Question 1: A rigid bar is supported by a 12 mm diameter pin at A and three 19 mm diameter cables at points B, C and D. The cables at B and D have a length of 3 m while the cable at C has a length of 5 m. All three cables are made of high strength steel with an elastic modulus of 200 GPa and yield stress of 800 MPa. The bar is loaded with a triangularly distributed load having a maximum intensity of 50 kN/m as shown.

Do the following:

[12 marks] (a) find the forces developed in each cable

[4 marks] (b) determine the corresponding displacement at the end of the bar (point E)

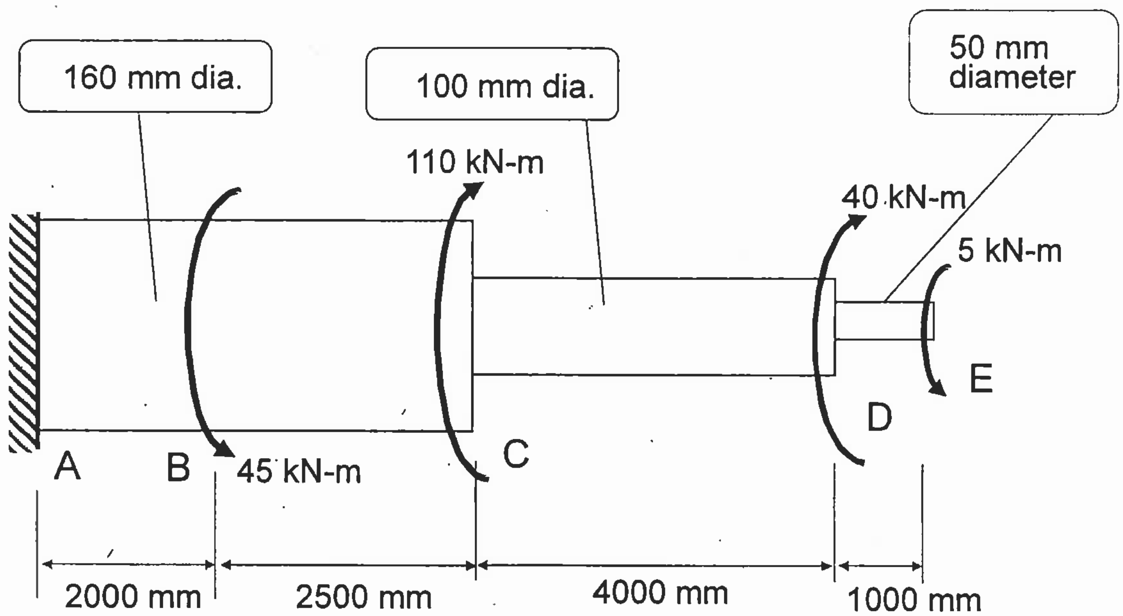
[4 marks] (c) find the corresponding shear stress in the pin at A given that the pin is loaded in double shear.



Question 2: A stepped steel shaft with G = 80 GPa and τ_y = 250 MPa is subjected to the torques shown. Dimensions (length and diameter) are also given.

[12 marks] (a) Determine the maximum shear stress in the shaft, and sketch the corresponding variation of shear stress along the shaft radius at this location.

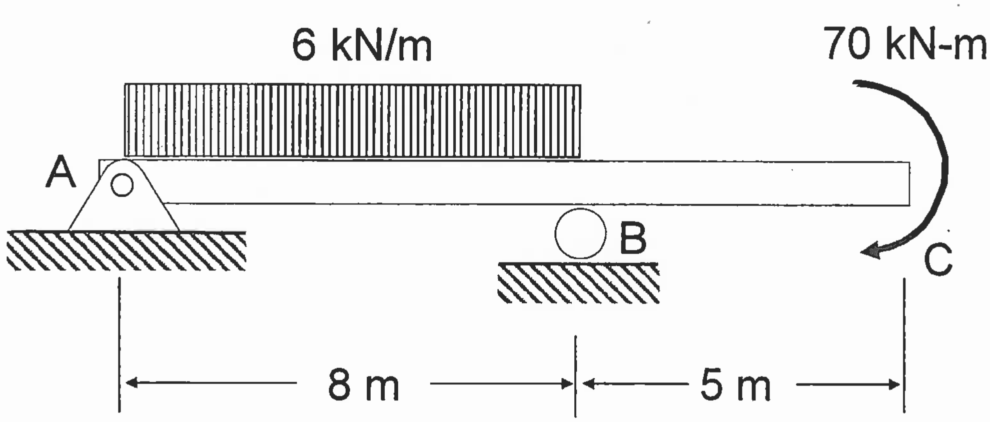
[8 marks] (b) Determine the rotation (in degrees) at the end of the shaft.



Question 3: [20 marks] A simply supported beam supports a uniformly distributed load of 6 kN/m acting over the simple span and a concentrated moment of 70 kN-m applied at the end of the overhang. The beam is a wide flange cross-section using steel with an allowable normal stress of 260 MPa and allowable shear stress of 75 MPa. The elastic modulus of steel is 200 GPa.

Determine the shear and moment throughout the beam as functions of x. This means that you need to give formula(s) for **V(x)** and **M(x)** along the length of the beam.

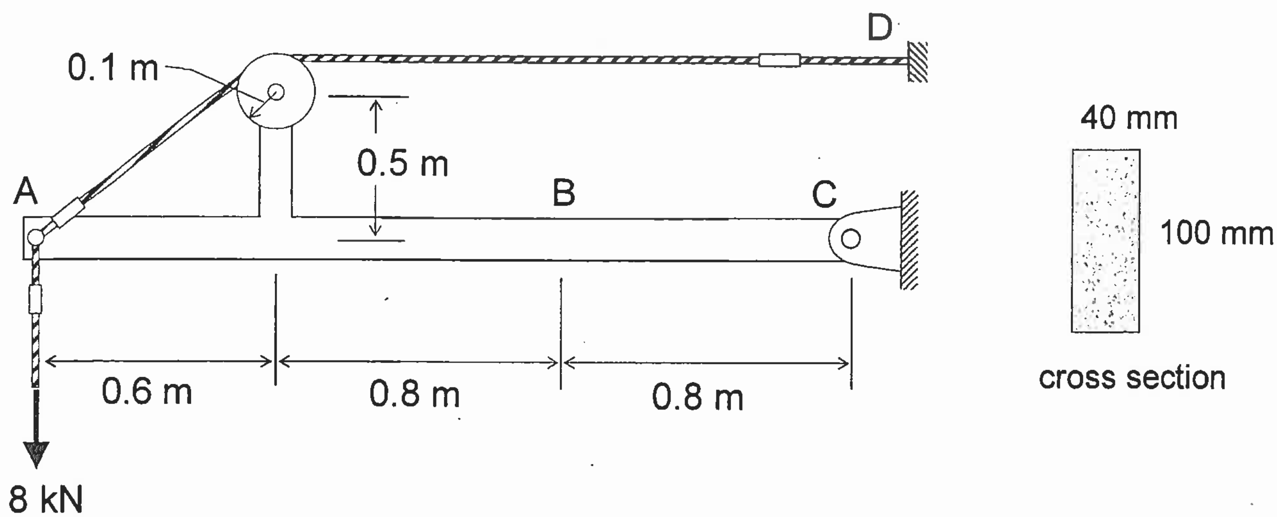
Next construct the shear force and bending moment diagrams. Remember to label points of maximum and negative bending moment, as well as any inflection points. Show your work by indicating exactly how you obtained your answer.



Remember that it is important you set this problem up correctly by calculating the correct reaction forces at the support(s).

No credit will be given for a solution using the principle of superposition, when combinations of existing solutions are used to find an answer.

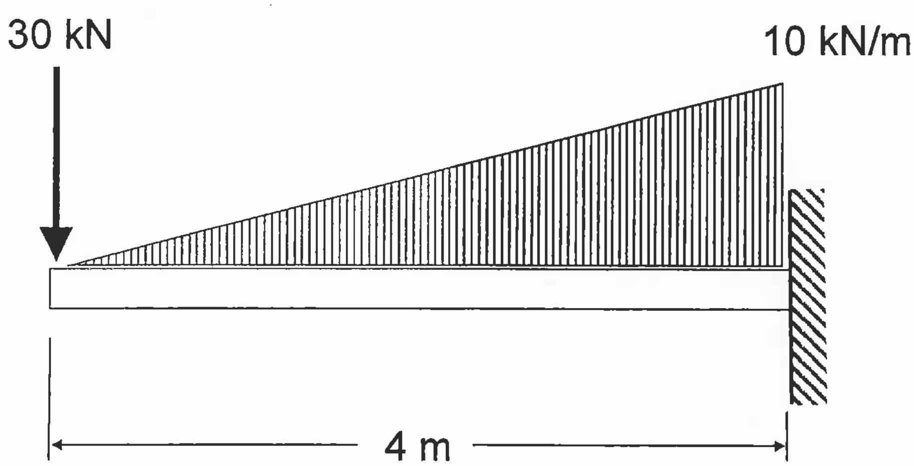
Question 4: A beam cable system supports a vertical load of 8 kN applied to the end of the cable
[20 marks] as shown below. The beam is rectangular in cross-section and is pinned at C. Compute the maximum and minimum values of normal stress and shear stress acting on the beam cross-section at location B. Show your answer with a sketch of the stress distribution (for both shear and normal stress), making sure to show maximum and minimum values of the stress.

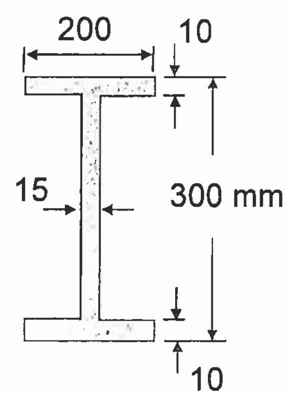


Question 5: A cantilevered beam (fixed at the base) supports a triangularly distributed load (with a maximum intensity of 10 kN/m) and a concentrated load of 30 kN as shown. The beam is a wide-flange section with the cross-section shown, and is made from steel having an allowable normal stress of 240 MPa and elastic modulus of 200 GPa.

[18 marks] (a) determine the deflection and slope at the free end of the beam using the method of integration.

[2 marks] (b) recognizing that computation of deflection is a lengthy process using the method of integration, explain how you would compute the deflection of this beam in a design situation.



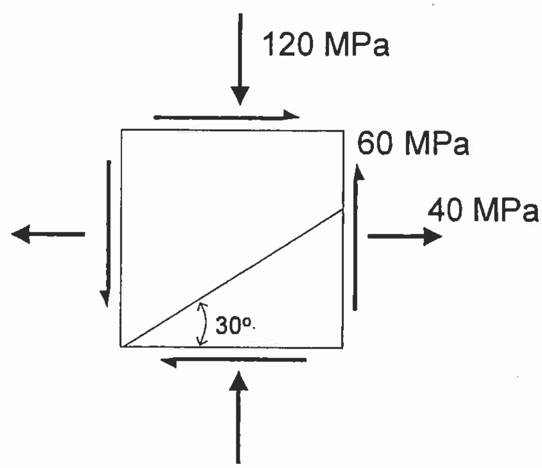


beam cross-section
(all dimensions in mm)

Question 6: [20 marks] For an element in a state of plane stress subjected to the normal and shear stresses shown below, use the Mohr's circle solution (*not* the transformation equations) to determine the following:

(a) the principal stresses and orientation of the principal planes, showing your answer on a properly oriented element.

(b) the maximum in-plane shear stress (and associated normal stresses) and orientation of the corresponding planes. Once again, show your answer on a sketch of a properly oriented element.

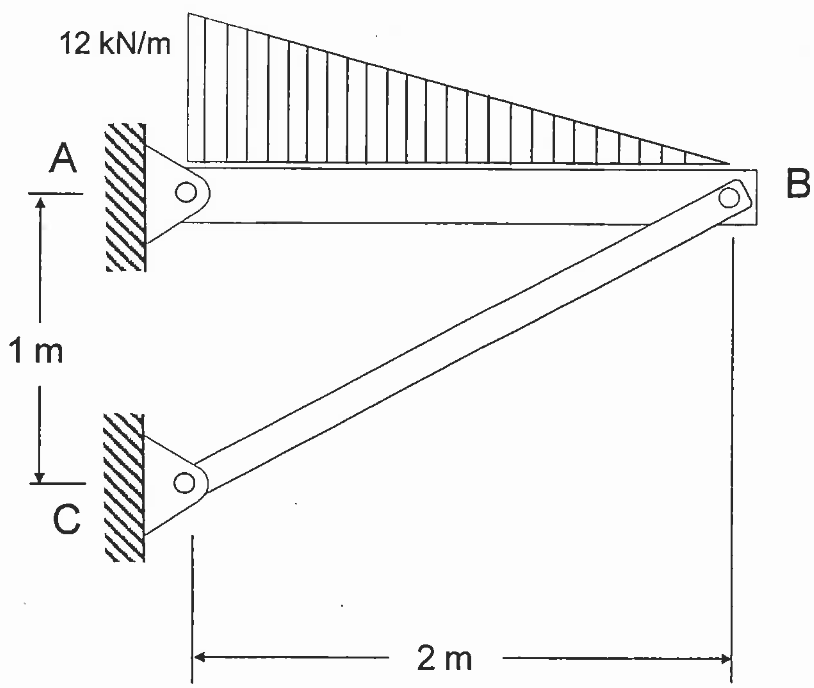


Note that credit will **only** be given for a **solution using Mohr's circle**. This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your **calculations** must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off! The stress transformation equations can only be used to check your answer.

Question 7: [20 marks]

The steel compression strut BC of the truss frame ABC shown below is a steel tube with an outside diameter of 50 mm and a wall thickness of 6 mm. The strut is pinned at both ends.

Determine the factor of safety against elastic buckling for the strut BC when a triangularly distributed load as shown is applied to the horizontal member AB. Consider in-plane buckling only. The steel used has an elastic modulus of $E = 200$ GPa and an allowable yield strength equal to 240 MPa.

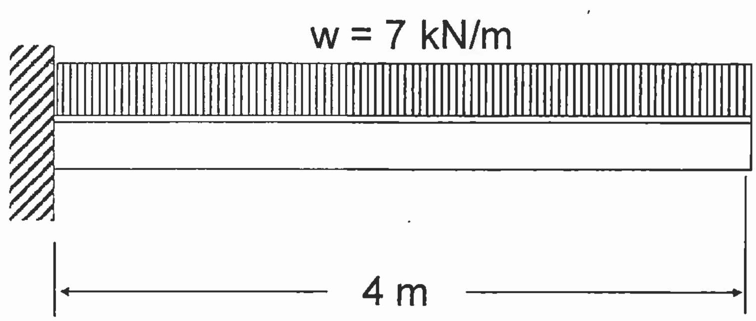


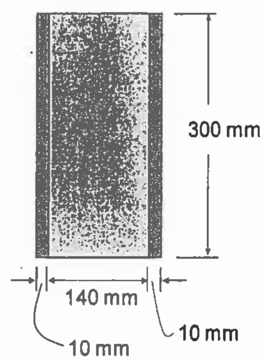
Note: $A_{circle} = \pi r^2$ and $I_{circle} = \pi r^4 / 4$

Question 8a: The cantilevered beam shown below is a composite member made of a rectangular timber section (140 mm wide by 300 mm high) reinforced by bonding 10 mm thick steel plates on either side of the wood beam as shown. The wood has an elastic modulus of 8 GPa and allowable normal stress of 6 MPa, while the steel has an elastic modulus of 200 GPa and allowable normal stress of 240 MPa.

[13 marks] Determine whether the composite beam can safely carry the applied loading as shown.

[2 marks] Explain why the steel plates need to be bonded to the wood section.

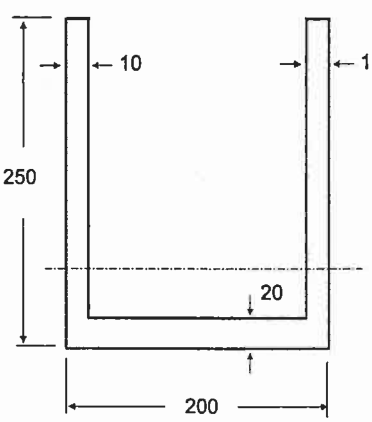




Beam Cross-Section

Question 8b: Find the plastic centroid depth (for full yielding) and corresponding plastic moment capacity for the channel cross section shown below. Assume the section is made of steel with a yield stress of 300 MPa and elastic modulus of 200 GPa. Bending is about the horizontal axis.

[5marks]



Beam Cross-Section
(all dimensions in mm)