

NATIONAL EXAMINATIONS DECEMBER 2011

04-BS-5 ADVANCED MATHEMATICS

3 Hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumption made.
2. Candidates may use one of the approved Casio or Sharp calculators. This is a Closed Book Exam. However, candidates are permitted to bring **ONE** aid sheet (8.5"x11") written on both sides.
3. Any five (5) questions constitute a complete paper. Only the first five answers as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme

1. 20 marks
2. 20 marks
3. (a) 5 marks ; (b) 9 marks ; (c) 3 marks ; (d) 3 marks
4. (A) 10 marks ; (B) 10 marks
5. 20 marks
6. (A) 10 marks ; (B) 10 marks
7. (a) 10 marks ; (b) 10 marks

1. Consider the following differential equation

$$(1-x^2)\frac{d^2y}{dx^2} - 5x\frac{dy}{dx} - 3y = 0$$

Find two linearly independent solutions about the ordinary point $x=0$.

2. Find the Fourier series expansion of the periodic function $f(x)$ of period $p=2\pi$.

$$f(x) = \begin{array}{ll} \frac{\pi}{2} & -\pi < x < -\frac{\pi}{2} \\ -x & -\frac{\pi}{2} < x < 0 \\ x & 0 < x < \frac{\pi}{2} \\ \frac{\pi}{2} & \frac{\pi}{2} < x < \pi \end{array}$$

3. Consider the following function where a is a positive constant

$$f(x) = \begin{array}{ll} \frac{a}{2} \cos(ax) & -\frac{\pi}{2a} < x < \frac{\pi}{2a} \\ 0 & \text{otherwise} \end{array}$$

(a) Compute the area bounded by $f(x)$ and the x -axis. Graph $f(x)$ against x for $a = 0.5$ and $a = 1$.

(b) Find the Fourier transform $F(\omega)$ of $f(x)$.

(c) Graph $F(\omega)$ against ω for the same two values of a mentioned in (a).

Explain what happens to $f(x)$ and $F(\omega)$ when a tends to infinity.

Note:
$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\exp(-i\omega x)dx$$

6.(A) The equation $x^3 + 3x^2 + 6x - 3 = 0$ has a root close to $x_0 = 0.5$. Use the following iterative formula twice to find a better approximation of this root. (Note: Carry seven digits in your calculations)

$$x_{i+1} = x_i - \frac{f(x_i)}{f^{(1)}(x_i)} - \frac{[f(x_i)]^2 f^{(2)}(x_i)}{2[f^{(1)}(x_i)]^3}$$

Hint: Let $f(x) = x^3 + 3x^2 + 6x - 3$. Note that $f^{(1)}(x)$ and $f^{(2)}(x)$ denote the first and second derivative of $f(x)$ respectively.

6.(B)(i) Let $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Show that the line joining the point (x_1, y_1) to the point (x_2, y_2) cuts the x-axis at

$$x_0 = \frac{x_2 y_1 - x_1 y_2}{y_1 - y_2}$$

(ii) Use the iterative formula developed above twice to find an approximate value of the root of the equation $3\sin(0.5x) + x^{0.5} = 0$ that lies between $x_1 = 4.5$ and $x_2 = 5.0$. (Note: Carry six digits in your computations)

7. The matrix $A = \begin{bmatrix} 9 & 3 & 9 \\ 3 & 5 & 5 \\ 9 & 5 & 26 \end{bmatrix}$ can be written as the product LL^T where

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \text{ and } L^T \text{ is the transpose of } L.$$

(a) Find L .

(b) Use the results obtained in (a) to solve the following system of three linear equations:

$$9x_1 + 3x_2 + 9x_3 = -9$$

$$3x_1 + 5x_2 + 5x_3 = 3$$

$$9x_1 + 5x_2 + 26x_3 = -22$$