NATIONAL EXAMS May 2010
07-Elec-B2 Advanced Control Systems

3 hours duration

NOTES:

1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of two calculators, a Casio or Sharp approved models.

3. This is a closed-book examination. Tables of Laplace and z-transforms are attached as page 4 and page 5.

4. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

5. All questions are of equal value.

1. Consider the automobile cruise control system below with,

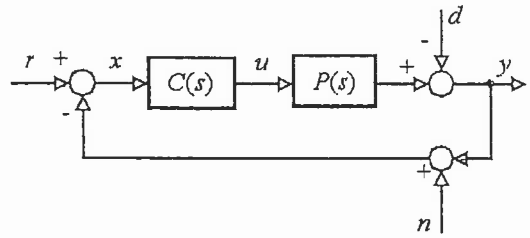
$$C(s) = \frac{5}{20s+1}, \quad P(s) = \frac{1}{(4s+1)^2}$$

(a) Determine the steady state error, $e = r - y$, when the grade is level, i.e., $d = 0$, the measurement noise, $n = 0$, and the setpoint speed is $r = 8$.

(b) Suddenly the grade increases such that $d = 3$, (with $r = 8$). Determine the new steady state error.

(c) Now, $r(t) = 8\sin(0.03t)$. Determine the steady state error, e.

(d) Re-design the controller, $C(s)$, such that the steady state error, e, is *zero* when $r(t)$ and $d(t)$ are arbitrary constants, measurement noise, $n = 0$, and the closed loop system is stable with a gain margin of at least 6dB.



2. Consider the system, $P(s) = \dfrac{16(\alpha s+1)}{s(s+2)^2}$.

(a) Find a state space model for the system.

(b) Justify the conditions under which the system controllable? observable?

(c) The system input and output are uniformly sampled with a sample period of h and the discrete input is applied to a zero order hold device. Determine the poles of the sampled data system as a function of h. Detailed calculations are not necessary.

3. Several experiments are conducted on an unknown plant:

When a step of magnitude 2 is applied to the input, the steady state output is 10.
When a sinusoid of frequency 8 rad/sec is applied, the phase lag at the output is 90°.
When a sinusoid of frequency 5 rad/sec is applied, the phase lag at the output is 15°.

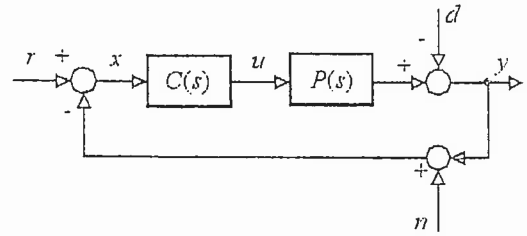
(a) Assume the system, $P(s)$, is second order system and has no finite zeros. Find the parameters of the second order model.

(b) For the model identified in (a) determine the maximum overshoot for a unit step input.

(c) Sketch the polar (Nyquist) plot for the identified model being careful to label axes, scales, and intercepts.

(d) Justify whether the system is stable or not when a controller, $C(s) = 1/s$, is cascaded with $P(s)$ in a negative (unity) feedback loop.

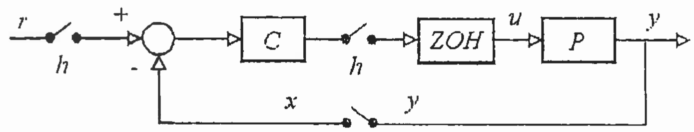
4. Consider the feedback system below with, $P(s) = \dfrac{4}{s^2+9}$.



(a) Determine a proper and stable $C(s)$ such that the transfer function that relates d to y is: $\dfrac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{(s+2)^2(s+1)}$. Recall that $C(s)$ is proper if the degree of the numerator is less than or equal to that of the denominator.

(b) Determine the transfer function that relates r to y and the transfer function that relates n to x.

5. Consider the sampled data system shown on the right. The input to the ZOH, the set-point, r, and the output, y, are uniformly sampled with a sample period of $h = 0.1$ with $C(z)$ and $P(s)$ given by,



$$C(z) = K, \quad P(s) = \frac{e^{-sh}}{s}$$

(a) Determine the discrete closed loop transfer function, $T(z)$, that relates $X(z)$ to $R(z)$.

(b) Determine the limiting value of K for stability.

(c) Consider replacing $C(z)$ by a statefeedback controller. Design the statefeedback controller (assuming states $y(k)$ and $y(k+1)$ are available for feedback) such that the closed loop poles are all at $z = 0.5$

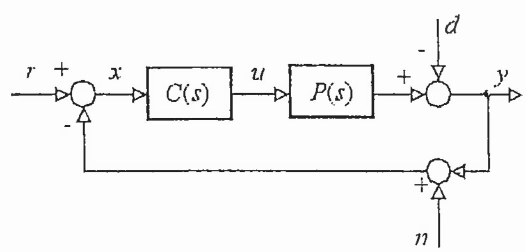
6. Consider the (continuous time) feedback system below with, $C(s) = K, \quad P(s) = \dfrac{2e^{-0.4s}}{s+1}$.

(a) Determine the range of K that results in closed loop stability.

(b) Determine the phase margin when $K = 1$ and sketch the associated Nyquist plot.

(c) The system is stable and operating with noise, $n(t) = 0$, disturbance, $d(t) = 1$, and set-point, $r(t) = 0$. Determine the tracking error, $e(t) = r(t) - y(t)$, as a function of K.

(d) It is necessary that the steady state error is zero for any arbitrary but constant value of d. Choose a new controller, $C(s)$, that ensures the steady state tracking error is zero *and* the closed loop system is stable with a gain margin of at least 4db.



Inverse Laplace Transforms	
$F(s)$	$f(t)$
$\frac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\frac{C+jD}{s+\alpha+j\beta}+\frac{C-jD}{s+\alpha-j\beta}$	$2e^{-\alpha t}\left(C\cos\beta t+D\sin\beta t\right)$
$\frac{A}{(s+\alpha)^{n+1}}$	$\frac{At^{n}e^{-\alpha t}}{n!}$
$\frac{C+jD}{\left(s+\alpha+j\beta\right)^{n+1}}+\frac{C-jD}{\left(s+\alpha-j\beta\right)^{n+1}}$	$\frac{2t^{n}e^{-\alpha t}}{n!}\left(C\cos\beta t+D\sin\beta t\right)$

Inverse z-Transforms	
$F(z)$	$f(nT)$
$\frac{Kz}{z-a}$	Ka^{n}
$\frac{\left(C+jD\right)z}{z-re^{j\varphi}}+\frac{\left(C-jD\right)z}{z-re^{-j\varphi}}$	$2r^{n}\left(C\cos n\varphi-D\sin n\varphi\right)$
$\frac{Kz}{\left(z-a\right)^{r}}$, $r=2,3\ldots$	$\frac{Kn\left(n-1\right)\ldots\left(n-r+2\right)}{\left(r-1\right)!a^{r-1}}a^{n}$

Table of Laplace and z-Transforms (h denotes the sample period)		
$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{\alpha h}}$
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos\beta h)}{z^2-2z\cos\beta h+1}$
$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin\beta h}{z^2-2z\cos\beta h+1}$
$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha h}\cos\beta h)}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha h}\sin\beta h}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$t\,f(t)$	$-\frac{dF(s)}{ds}$	$-zh\frac{dF(z)}{dz}$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$	$F(ze^{\alpha h})$