

NATIONAL EXAMINATIONS MAY 2010

04-BS-2

PROBABILITY AND STATISTICS

2 HOURS DURATION

**NOTES:**

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumption made.
2. “Closed Book” – no-aids other than
  - (i) A Casio or Sharp approved calculator
  - (ii) ONE hand-written information sheet (8.5”x11”), filled on both sides.
3. Any 5 questions constitute a complete paper. Only 5 questions will be marked.
4. All questions are of equal value.
5. Statistical tables of the normal, t, chi-square and F distributions are provided.

**Marking Scheme**

- 1.(a) 5 marks (b) 5 marks (c) 5 marks (d) 5 marks
- 2.(a) 5 marks (b) 5 marks (c) 5 marks (d) 5 marks
- 3.(A) 5 marks (B) 5 marks (C) (a) 5 marks (b) 5 marks
- 4.(a) 7 marks (b) 7 marks (c) 6 marks
- 5.(A) (a) 5 marks (b) 5 marks (B) (a) 5 marks (b) 5 marks
6. 20 marks
- 7.(a) 10 marks (b) 10 marks
- 8.(a) 5 marks (b) 5 marks (c) 5 marks (d) 5 marks

1. The weight  $W$  of the 60cm×60cm patio stones sold by Delightful Gardens is a normally distributed random variable with mean and standard deviation equal to 22.6kgs and 0.8kgs respectively.

- (a) Mr. Chancey buys one of this type of patio stones in order to replace a similar one that was broken in his backyard. What is the probability that the new patio stone will weigh less than 22.0kgs? Draw the probability density function of  $W$ , neatly and clearly, and indicate the area that corresponds to this probability.
- (b) Compute (i) the lower quartile and (ii) the upper decile of the probability distribution of  $W$ . Explain, clearly and neatly, the meaning of these quantities.
- (c) Let  $M$  represent the mean weight of a random sample of 16 patio stones. (i) Find the mean and standard deviation of  $M$ . (ii) Write down the probability density function of  $W$  and  $M$ . (iii) Draw the probability density function of  $W$  and  $M$  on the same diagram. (iv) Compute the probability that  $M$  exceeds 22.5kgs.
- (d) Let  $T$  be the sum of the weights of 25 patio stones. Find  $E(T)$  and  $\text{Var}(T)$ . Then compute the probability that  $T$  exceeds 560.0kgs. Draw the probability density function of  $T$ , neatly and clearly, and indicate the area that corresponds to this probability.

2. Extensive data gathered by the manager of the Water Services Division of a large municipality revealed that the number of calls received by the maintenance department of the division follows the Poisson law with an average of 3.5 calls per hour.

- (a) Compute the probability that the maintenance department receives more than 2 but fewer than 7 calls in a given hour.
- (b) Compute the probability that the maintenance department receives more than 5 calls in a period of two hours.
- (c) Use an appropriate approximation to find the probability that the maintenance division receives at least 90 calls during a period of 24 hours.
- (d) Find the probability that the maintenance division receives 3 calls in a given hour and 5 calls in the following hour.

3.(A) Mrs. Moonlight bought a box of 12 fluorescent light bulbs from CheapoMart. Unknown to Mrs. Moonlight 4 of the light bulbs contained in the box are substandard.

- (a) Mrs Moonlight randomly selects 6 bulbs from this box and installs them in her chandelier. What is the probability that at least four are standard?
- (b) Assume that a sample of five bulbs is randomly selected from the box. Let  $X$  denote the number of substandard bulbs in the sample. Find the probability distribution of  $X$ . Then find the mean and variance of  $X$ .

3.(B) Sixty percent of the calls received by the maintenance department of Happy House Heating (HHH) during the winter months concern the failure of the electric motor used in the heating system.

- (a) Find the probability that in a random sample of 15 calls received by the maintenance department of HHH more than 6 but fewer than 11 concern the electric motor.
- (b) Use an appropriate approximation to find the probability that in a random sample of 700 calls more than 400 concern the electric motor.

4. The following data pertain to the daily output  $X$ , in tonnes, of a continuous chemical process over a period of 14 days

$$\sum X = 285.50; \quad \sum X^2 = 5830.16$$

- (a) Find the 99% confidence limits of (i) the true mean and (ii) the true standard deviation of the daily output. Assume that  $X$  is a normally distributed random variable.
- (b) Test the hypothesis that the mean daily output is not significantly different from 20.6 tonnes. Let  $\alpha = 0.05$ .
- (c) Test the hypothesis that the true standard deviation is not significantly different from 0.60 tonnes. Let  $\alpha = 0.05$ .

5.(A) A random sample of 625 bricks manufactured by Coliseum Works using a new process yielded a mean compressive strength equal to 5,550psi and a variance equal to 40,000psi.

- (a) The owner of Coliseum Works claims that the compressive strength of the bricks manufactured by this new process is not significantly different from 5,560psi. Do the data support her claim? Let  $\alpha = 0.05$ .
- (b) How large should the sample be if we wish to know the true mean compressive strength with a maximum error of 4psi and 99% confidence?

5.(B) A survey carried out on behalf of Proficient Hardware revealed that in a random sample of 800 customers 650 use Proficient's credit card to pay for their purchases.

- (a) Find a 90% confidence interval of the true proportion of customers who use Proficient's credit card to pay for their purchases.
- (b) What should the size of the sample be if we wish to know the true proportion with a maximum error of 0.03 and 99% confidence?

6, The quality of mattresses manufactured by ABX are classified as superior, good, average and inferior by the Production and Quality Control department. Past performance of the plant shows that 70% of the production is classified as superior, 15% as good, 10% as average and 5% as inferior. As a check on last month's run of the plant 840 mattresses were randomly selected and after careful examination yielded the following classification

QUALITY	Superior	Good	Average	Inferior
FREQUENCY	600	160	70	10

Test the hypothesis that there was no change in the quality of production last month. Let  $\alpha=0.05$ .

7. The following results represent a summary of the tests carried out by the Quality Control Department of Senior Motors Corporation (SMC) to determine the lifetime of high intensity light bulbs manufactured by two different companies. Originally sixteen bulbs from each manufacturer were tested. However, due to some clerical errors, five results had to be discarded. The remaining results of these tests were as follows:

	Manufacturer A	Manufacturer B
Sample size	$n_A = 13$	$n_B = 14$
Sample Mean (hours)	$m_A = 3,000$	$m_B = 3,120$
Sample Standard Deviation (hours)	$s_A = 90$	$s_B = 115$

- (a) Test the hypothesis that the variability of the lifetime of bulbs obtained from Manufacturer A is not significantly different from that obtained from Manufacturer B. Let  $\alpha=0.05$ . State any assumptions you need to make.
- (b) Test the hypothesis that the mean lifetime of bulbs obtained from Manufacturer A is not significantly different from that obtained from Manufacturer B. Let  $\alpha=0.05$ .

8. The following data represent observations of the stack loss Y of nitric acid, in grammes per cubic metre, corresponding to the temperature X, in degrees Celsius, of the cooling system of a plant preparing nitric acid by the oxidation of ammonia with air.

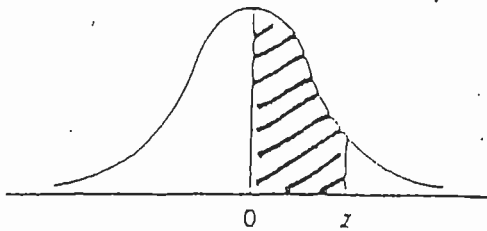
$$\sum_{i=1}^n X_i = 614.0 \quad ; \quad \sum_{i=1}^n X_i^2 = 12,820.0 \quad ; \quad \sum_{i=1}^n Y_i = 78.7;$$

$$\sum_{i=1}^n Y_i^2 = 228.2 \quad ; \quad \sum_{i=1}^n X_i Y_i = 1662.6 \quad ; \quad n=30$$

- (a) Compute  $Cov(X, Y)$  and the coefficient of correlation r.
- (b) Test the hypothesis that the true coefficient of correlation  $\rho$  is not significantly different from zero Let  $\alpha = 0.05$ .

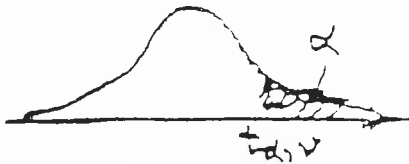
- (c) It is believed that  $Y$  and  $X$  are related by an equation of the form  $Y = \beta_0 + \beta_1 X + \varepsilon$ . Write down the normal equations of the least squares line and then compute the estimates  $b_0$  and  $b_1$  of  $\beta_0$  and  $\beta_1$  respectively.
- (d) Compute the error sum of squares. Then use this information to find the 95% confidence limits of  $\beta_1$ .

NORMAL DISTRIBUTION TABLE



$$F(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-t^2/2} dt$$

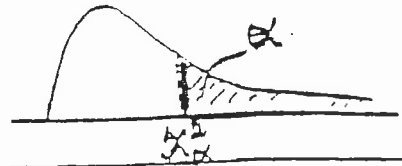
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1405	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2957	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3952	.3980	.3997	.4015
1.3	.4032	.4049	.4056	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4255	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4905	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990



t - DISTRIBUTION

v	$\alpha = 0.100$	$\alpha = 0.050$	$\alpha = 0.025$	$\alpha = 0.010$	$\alpha = 0.005$	v
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.523	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.835	2.262	2.821	3.250	9
10	1.372	1.817	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
30	1.282	1.645	1.950	2.326	2.576	30

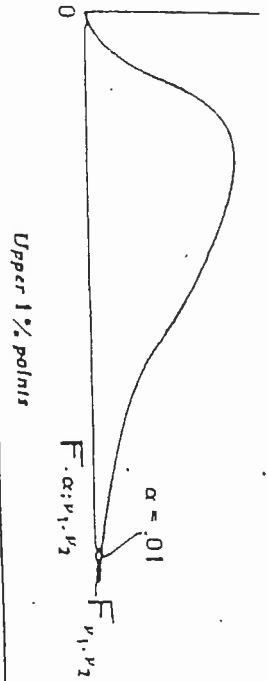
THE CHI-SQUARE DISTRIBUTION



v	Probability that chi-square value will be exceeded							
	.995	.990	.975	.950	.050	.025	.010	.005
1	---	---	---	.004	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	11.07	12.83	15.09	16.75
6	.58	.87	1.24	1.64	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	16.92	19.02	21.57	23.59
10	2.16	2.56	3.25	3.94	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	22.36	24.74	27.69	29.82
14	4.07	4.66	5.53	6.57	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	35.17	38.08	41.64	44.19
24	9.89	10.86	12.40	13.85	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	41.34	44.46	48.23	50.99
29	13.12	14.26	16.05	17.71	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	55.76	59.34	63.69	66.77
50	27.99	29.71	32.26	34.76	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	79.08	83.30	88.33	91.9
70	43.28	45.44	48.76	51.74	90.53	95.02	100.43	104.2
80	51.17	53.54	57.15	60.39	101.88	106.63	112.33	116.3
90	59.20	61.75	65.65	69.13	113.14	118.14	124.12	128.3
100	67.33	70.06	74.22	77.93	124.34	129.56	135.81	140.1



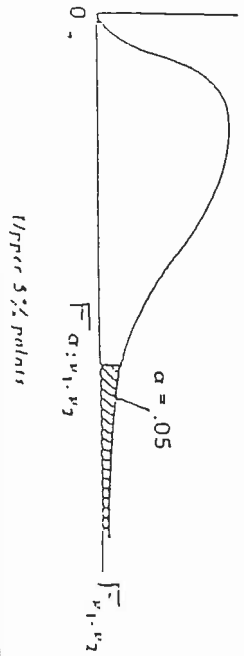
F - DISTRIBUTION



	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$	
40:1																				
91.50	4999.5	3403	5675	5764	5839	5928	5982	6032	6056	6106	6137	6209	6215	6261	6287	6313	6339	6339	6366	
34.12	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.49	99.50	
21.20	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.22	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13	26.11	
16.26	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.53	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46	13.45	
13.75	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02	8.88	
12.25	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88	6.85	
10.56	9.35	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65	5.63	
10.04	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31	4.11	
9.65	7.35	6.55	5.92	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91	3.60	
9.33	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60	3.06	
9.07	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.19	4.08	3.96	3.88	3.80	3.72	3.64	3.56	3.47	2.75	
8.86	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.00	3.89	3.78	3.69	3.60	3.51	3.43	3.35	3.27	2.65	
8.68	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.84	3.73	3.62	3.51	3.43	3.35	3.27	3.18	3.10	2.49	
8.53	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.69	3.58	3.47	3.37	3.29	3.21	3.12	3.04	2.96	2.42	
8.40	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.58	3.47	3.36	3.26	3.18	3.10	3.00	2.92	2.84	2.31	
8.29	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.37	3.26	3.16	3.08	3.00	2.92	2.84	2.76	2.23	
8.18	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.61	3.52	3.41	3.30	3.20	3.10	3.02	2.94	2.86	2.78	2.70	2.17	
8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.12	3.03	2.94	2.86	2.78	2.70	2.62	2.54	2.13	
8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.06	2.96	2.88	2.80	2.72	2.64	2.56	2.48	2.10	
7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	3.01	2.91	2.83	2.75	2.67	2.59	2.51	2.43	2.06	
7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.96	2.86	2.78	2.70	2.62	2.54	2.46	2.38	2.03	
7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.92	2.82	2.74	2.66	2.58	2.50	2.42	2.34	2.01	
7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.88	2.78	2.70	2.62	2.54	2.46	2.38	2.30	1.97	
7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.95	2.84	2.74	2.66	2.58	2.50	2.42	2.34	2.26	1.93	
7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.92	2.81	2.71	2.63	2.55	2.47	2.39	2.31	2.23	1.90	
7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.89	2.78	2.68	2.60	2.52	2.44	2.36	2.28	2.20	1.87	
7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.76	2.66	2.58	2.50	2.42	2.34	2.26	2.18	1.85	
7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.73	2.63	2.55	2.47	2.39	2.31	2.23	2.15	1.82	
7.51	5.38	4.50	4.01	3.69	3.46	3.29	3.16	3.06	2.97	2.83	2.72	2.62	2.54	2.46	2.38	2.30	2.22	2.14	1.81	
7.48	5.38	4.50	4.01	3.69	3.46	3.29	3.16	3.06	2.97	2.83	2.72	2.62	2.54	2.46	2.38	2.30	2.22	2.14	1.81	
7.41	5.31	4.43	3.94	3.62	3.39	3.22	3.09	2.99	2.90	2.76	2.65	2.55	2.47	2.39	2.31	2.23	2.15	2.07	1.74	
7.31	5.18	4.31	3.82	3.50	3.27	3.10	2.97	2.87	2.78	2.64	2.53	2.43	2.35	2.27	2.19	2.11	2.03	1.95	1.62	
7.08	4.98	4.11	3.62	3.30	3.07	2.90	2.77	2.67	2.58	2.44	2.33	2.23	2.15	2.07	1.99	1.91	1.83	1.75	1.42	
6.85	4.79	3.95	3.46	3.14	2.91	2.74	2.61	2.51	2.42	2.28	2.17	2.07	1.99	1.91	1.83	1.75	1.67	1.59	1.26	
6.83	4.79	3.95	3.46	3.14	2.91	2.74	2.61	2.51	2.42	2.28	2.17	2.07	1.99	1.91	1.83	1.75	1.67	1.59	1.26	
6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.07	1.97	1.89	1.81	1.73	1.65	1.57	1.49	1.16	



F - DISTRIBUTION



	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.2	251.1	251.1	252.2	253.3	254.3
18.51	19.00	19.15	19.23	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.47	19.48	19.49	19.50
10.13	9.55	9.28	9.12	9.01	8.94	8.85	8.81	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.59	8.57	8.55	8.53
7.71	6.94	6.39	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.72	5.69	5.65	5.61
6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.46	4.41	4.40	4.36
5.99	5.14	4.76	4.51	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.77	3.74	3.70	3.67
5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.34	3.30	3.27	3.23
5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.04	3.01	2.97	2.93
5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.83	2.79	2.75	2.71
4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.66	2.62	2.58	2.54
4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.53	2.49	2.45	2.40
4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.43	2.38	2.34	2.30
4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.34	2.29	2.25	2.21
4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.27	2.22	2.18	2.14
4.54	3.68	3.28	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.21	2.21	2.16	2.12	2.07
4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.20	2.16	2.16	2.11	2.06	2.01
4.45	3.59	3.20	2.96	2.81	2.70	2.62	2.55	2.49	2.44	2.38	2.30	2.23	2.19	2.15	2.11	2.11	2.06	2.02	1.97
4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.45	2.41	2.34	2.27	2.20	2.16	2.12	2.08	2.08	2.03	1.99	1.94
4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.24	2.17	2.13	2.09	2.05	2.05	2.00	1.96	1.91
4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.44	2.39	2.35	2.28	2.20	2.13	2.09	2.05	2.01	2.01	1.96	1.92	1.87
4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.11	2.07	2.03	1.99	1.99	1.94	1.90	1.85
4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.08	2.04	2.00	1.96	1.96	1.91	1.87	1.82
4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.06	2.02	1.98	1.94	1.94	1.89	1.85	1.80
4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.04	1.98	1.94	1.90	1.90	1.85	1.81	1.76
4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.02	1.96	1.92	1.88	1.88	1.83	1.79	1.74
4.23	3.39	2.98	2.74	2.58	2.47	2.38	2.32	2.26	2.22	2.14	2.07	1.99	1.93	1.89	1.85	1.85	1.80	1.76	1.71
4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.21	2.13	2.06	1.98	1.92	1.88	1.84	1.84	1.79	1.75	1.70
4.20	3.34	2.95	2.71	2.55	2.44	2.35	2.29	2.24	2.19	2.12	2.04	1.97	1.90	1.86	1.82	1.82	1.77	1.73	1.68
4.18	3.33	2.93	2.70	2.54	2.43	2.34	2.28	2.22	2.18	2.10	2.03	1.95	1.89	1.85	1.81	1.81	1.76	1.72	1.67
4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.94	1.87	1.83	1.79	1.79	1.74	1.70	1.65
4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.77	1.73	1.69	1.69	1.64	1.60	1.55
4.00	3.15	2.76	2.53	2.37	2.26	2.17	2.10	2.04	1.99	1.92	1.84	1.76	1.69	1.65	1.61	1.61	1.56	1.52	1.47
3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.59	1.55	1.51	1.51	1.46	1.42	1.37
3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.48	1.44	1.44	1.39	1.35	1.30
3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.48	1.44	1.44	1.39	1.35	1.30