

National Exams May 2010
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
 2. NO CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
 3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
 4. All questions are of equal value.
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Marking Scheme:

1. 20 marks
2. 20 marks
3. 20 marks
4. (a) 6 marks, (b) 6 marks, (c) 8 marks
5. 20 marks
6. (a) 7 marks, (b) 7 marks, (c) 6 marks
7. 20 marks
8. 20 marks

1. Find the solution, $y(x)$, of the differential equation

$$y'' + 9y = \sec 3x,$$

$y'(0) = 0$, $y(0) = 1$. Note that ' denotes differentiation with respect to x .

2. Find the general solution, $y(x)$, of the differential equation

$$2x^2y'' - xy' - 2y = 3x^4.$$

Note that ' denotes differentiation with respect to x .

3. Find the line tangent to the intersection of the surfaces

$$3x^2 + 2y^2 - 2z = 1$$

and

$$x^2 + y^2 + z^2 - 4y - 2z + 2 = 0$$

at the point $(1, 1, 2)$.

4. Consider the matrix

$$A = \begin{pmatrix} 3 & 2 & 0 \\ 0 & 1 & 0 \\ -10 & -4 & -2 \end{pmatrix}$$

- (a) Show that $\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ is an eigenvector of A and find the associated eigenvalue.
(b) Show that 3 is an eigenvalue of A and find an associated eigenvector.
(c) Solve the linear system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ for the function $\mathbf{x}(t)$.

5. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where

$$\mathbf{F}(x, y, z) = (3x - y)\mathbf{i} + (x + 3y)\mathbf{j} + 2z\mathbf{k},$$

S is the surface of the region bounded by the plane $z = 4$ and the paraboloid $z = x^2 + y^2$.

6. Consider the two lines defined as follows:

$$\begin{array}{lll} x = 3 - 2t, & y = 3, & z = 3 - t, \text{ (parameter } t\text{);} \\ x = s, & y = 1 - 2s, & z = 2 + s, \text{ (parameter } s\text{).} \end{array}$$

- (a) Determine whether or not the two lines intersect, and if so, find the point of intersection.
- (b) Find a third line orthogonal to both lines.
- (c) Is there a plane containing both lines? If so, find an equation for that plane.

7. Find the maximum and minimum values of $f(x, y, z) = 3x + 2y^2 + z$ over the ellipsoid $3x^2 + y^2 + z^2 = 1$.

8. Find the volume of the solid region above the plane $z = -4$ and below the paraboloid $z = 4 - 2x^2 - 2y^2$.