National Exams December 2010

98-Phys-A7, Optics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate should include in the answer clear statements of the interpretation and any assumptions made.
2. This is a **CLOSED BOOK EXAM**.
3. Candidates may use one of two calculators, the Casio or Sharp **approved models**.
4. Answers to question 1 plus any **three** of questions 2 to 6 constitutes a complete exam paper.
5. Answer question 1 in the space provided on the exam paper.
6. The first three questions as they appear in the answer book will be marked.
7. Each question is of equal value. Question 1 is mandatory.

1. Answer question one in the space provided. A phrase, or a diagram and a phrase, is all that is required in most cases. [25 marks, one mark for each letter]

a) This exam is about light and optics. Define *light*.

b) Define *optics*.

c) Define *geometrical optics*.

d) Define *physical optics*.

e) What problems with *geometrical optics* led to the development of *physical optics*?

f) Define *plane of incidence*.

g) State the *law of reflection*.

h) State the *law of refraction*.

i) Complete any two rows of the following table:

		wavelength range (nm)	frequency range (Hz)
	UV light		
	red light		
	blue light		
	IR light		

Maxwell's equations are:

$$\nabla \cdot D = \rho_f$$
$$\nabla \cdot B = 0$$
$$\nabla \times E = -\frac{\partial B}{\partial t}$$
$$\nabla \times H = J_f + \frac{\partial D}{\partial t}$$

j) Define and give SI units for the E that appears in Maxwell's equations.

k) Define and give SI units for the B that appears in Maxwell's equations.

l) Define D in terms of E. Assume a linear, homogeneous, isotropic dielectric.

m) Define H in terms of B. Assume a linear, nonferromagnetic, homogeneous, isotropic medium

n) Over what frequency range are Maxwell's equations valid?

o) Maxwell's equations can be solved under certain approximations to find that E and B are transverse plane waves. What are the approximations?

p) Describe a transverse plane wave.

q) For most problems an analysis in terms of monochromatic plane waves is not restrictive. Why?

r) Write an equation for the *Poynting vector* and give the SI units for the quantity the Poynting vector represents.

s) What is the *refractive index of a material* defined to be equal to?

t) What defines the *direction of polarization* of light?

u) What is the difference between *interference* and *diffraction*?

v) What does the concept of *spatial coherence* deal with?

w) What does the concept of *temporal coherence* deal with?

x) Define *dispersion* as it relates to the refractive index.

y) At what speed does light appear to propagate?

2. [25 marks total]
A thin film of MgF_2 of index of refraction n = 1.40 is deposited on a glass with refractive index > 1.40 so that the glass-film system is antireflecting at a wavelength of 600 nm under normal incidence. Let the plane of incidence be the *x-y* plane and let the normal to the thin film be the y direction. Follow the steps indicated

(i) Write expressions for TE and TM polarized plane waves that are travelling at an angle of θ with respect to the y axis. Be sure to include in your answer the E and H (or B) fields and to draw the coordinate system. [6 marks]

(ii) Assume normal incidence and that the reflectivity of the air-MgF_2 interface is r_1 and that the reflectivity of the MgF2-glass interface is r_2. Using the expressions from (i) write an expression for the E field in an observation plane. **Ignore multiple reflections.** Argue the condition for minimum reflectance from the air-MgF_2-glass system and hence determine the thickness of the film for the reflection at normal incidence to be a minimum. Provide a sketch of the system, trace rays through the system, and label relevant quantities (coordinate system, interfaces, reflectivities, distances, incident ray, reflected rays, observation plane, ...). [5 marks]

(iii) Now assume an angle of incidence of 30 deg and that the reflectivity of the air-MgF_2 interface is r_1 and that the reflectivity of the MgF_2-glass interface is r_2. Using the expressions developed in (i) write an expression for the E field in an observation plane. **Ignore multiple reflections**. Argue the condition for minimum reflectance from the air-MgF_2-glass system and hence determine what wavelength is minimally reflected. Draw a sketch of the optical system and trace relevant rays (i.e., calculate the angles) through the system. Label the angles for the relevant rays that are required to solve the problem and label relevant quantities (coordinate system, interfaces, reflectivities, distances, incident ray, reflected rays, observation plane, ...). [8 marks]

(iv) Repeat the analysis of (ii) and of (iii) for the case that the refractive index of the glass is < 1.40 Compare to your answers found in (ii) and (iii) and explain the origin of any differences. [6 marks]

3. [25 marks total]
Design a 0.5 m, *f*/5 monochromator to provide a spectral resolution of 0.1 nm at 500 nm in first order for light from an extended, isotropic source. Draw a sketch of your design, provide dimensions, and describe the purpose of each element in your design. Use a simple lens to couple light into the monochromator and specify the focal length and diameter of this lens.

4. [25 marks total]

(a) Sketch the reflectance and transmittance as a function of the angle of incidence for (i) external reflection and (ii) internal reflectance for linearly polarized light. Remember that there are two polarizations. Define as requested, and label curves, the critical angles, the polarizing angles (Brewster's angles), and place scales on the ordinate and on the abscissa.

(i) Define *external reflection* and sketch reflectance and transmittance on the same graph. [5 marks]

(ii) Define *internal reflection* and sketch reflectance and transmittance on the same graph. [5 marks]

(b) Repeat (a), but this time for circularly polarized light. [5 marks]

(c) **Derive** an expression for $T(\theta)$ in terms of $t(\theta)$, n_1, n_2, and θ where θ is the angle of incidence. Define clearly the variables and explain your reasoning at each step. [10 marks]

5. [25 marks total]

(a) Sketch the optical system that is described by the system matrix. Label the elements and indicate distances [6 marks].

$$\begin{bmatrix} 1 & 16.67 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -0.06 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -0.05 & 1 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.833 & 0 \\ -0.110 & -1.2 \end{bmatrix}$$

(b) The system matrix is used with a column vector of length 2. State clearly what the two elements of the column vector represent [2 marks].

(c) Explain how one can determine from a system matrix the location of an image plane [2 marks].

(d) Use the system matrix to trace two useful rays through the optical system [5 marks].

(e) [10 marks] A beam of linearly polarized light is transformed into right circularly polarized light upon transmission through a crystal that is 1 mm thick. (i) Calculate the possible differences in the refractive indices for the two directions in the crystal. [4 marks] (ii) Sketch and label the optical system required to convert the linearly polarized light to right circularly polarized light. [4 marks] (iii) How would the system be modified to create left circularly polarized light? [2 marks]

6. [25 marks total]

(a) State the Huygens-Fresnel Principle and draw a sketch to indicate how the field at an observer is found. [5 marks]

(b) Find the far-field for plane wave illumination of a single slit of width $2d$. Allow for the case that the illumination is not at normal incidence to the slit. [5 mark].

(c) Two lights are 1 m apart. At what distance from the lights is an observer able to just resolve the two lights? Assume an eye diameter of 5 mm and a wavelength of 500 nm. [5 marks]

(d) A grating of width 2 cm is expected to provide a first order resolution of 0.1 nm anywhere in the spectral range of 400 nm to 700 nm. Determine the minimum number of grooves required. If the far field pattern is observed with a lens of focal length 50 cm, what is the linear separation of a 0.1 nm interval in the vicinity of 500 nm? [5 marks]

(e) The energy density of light of wavelength = 660 nm is reduced to one eighth of its original value by propagation through 200 cm of sea water. What is the absorption coefficient for light of this wavelength? At what depth is light of wavelength = 660 nm reduced to 1% of its original value? [5 marks].

Appendix

The intensity in the far-field as a function of the angle θ from the normal of the diffraction grating of N lines, line spacing of a, and line width of b, for illumination with a plane wave with $k = 2\pi/\lambda$ and an angle of incidence of θ_i is

$$I(\theta) = I_o \left[\frac{\sin(\beta)}{\beta} \right]^2 \left[\frac{\sin(N\alpha)}{\sin(\alpha)} \right]^2$$

$$\beta = \frac{kb}{2}(\sin\theta_i + \sin\theta)$$

$$\alpha = \frac{ka}{2}(\sin\theta_i + \sin\theta)$$

double angle formulae:

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = 2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

The resolution R and the dispersion for a grating with N lines and order m are

$$R = \frac{\lambda}{\Delta\lambda} = mN$$

$$D = \frac{m}{a\cos\theta}$$

For a circular lens of diameter D and image distance s, the full width of the central diffraction maximum between zeros is $2.44\ s\lambda/D$.

$$E(x, y, z) = \frac{ik}{2\pi}\frac{e^{ikz}}{z} e^{i\frac{k}{2z}(x^2 + y^2)} \iint E(x_s, y_s, 0) e^{-i\frac{k}{z}(xx_s + yy_s)} \mathrm{d}x_s \, \mathrm{d}y_s$$

zone plate radii $R_m = (m\ r_o\ \lambda)^{0.5}$

$$(1 + \varepsilon)^{\xi} = 1 + \frac{\xi}{1!}\varepsilon + \frac{(\xi)\times(\xi-1)}{2!}\varepsilon^2 + \ldots$$

$\gamma = \frac{1}{2}\, k\, D \sin\theta$ The zeros for $J_1(\gamma)$ occur for γ = 0.0, 3.832, 5.136, 7.016, 8.417, 10.173, 11.620, 13.324, . . .

$$\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The translation, refraction at a spherical interface, thin lens, and spherical mirror matrices are listed below.

$$\begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ \frac{n_1 - n_2}{R n_2} & \frac{n_1}{n_2} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$$

THE END