National Exams May 2009

98-Comp-B6 Computer Control and Robotics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. This is a CLOSED BOOK EXAM.
   Any non-communicating calculator is permitted.

3. FIVE (5) questions constitute a complete exam paper.
   The first five questions as they appear in the answer book will be marked.

4. Each question is of equal value.

1. (20 marks)

Let the Z-transform of $f(k)$ be denoted $F(z)$.

a) Find the Z-transform of the following sampled signal.

$$f(k) = (0.6)^k - 3(0.3)^k, \qquad k = 0,1,2,...$$

b) Compute the inverse Z-transform of

$$F(z) = \frac{3z^2 - 4z}{z^2 - 0.7z + 0.12}$$

2. (20 marks)

Let the Z-transform of $f(k)$ be denoted $F(z)$. Use the Z-transform method to find the solution of the following difference equation.
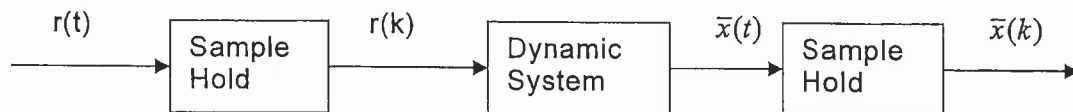
$$y(k+2) - 0.7y(k+1) + 0.12y(k) = 0, \qquad k = 0,1,2,...$$

The initial conditions are $y(1) = -1.9, \quad y(0) = 3$.

Hint: Use the property $Z\{f(k+n)\} = z^n F(z) - z^n f(0) - z^{n-1} f(1) - \cdots - zf(n-1)$.

3. (20 marks)

A dynamic system has the following continuous time model.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix} r(t)$$

r(t) → | Sample Hold | → r(k) → | Dynamic System | → $\bar{x}(t)$ → | Sample Hold | → $\bar{x}(k)$ →

Write the discrete time equation that describes this system in the form,

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = A \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + Br(k)$$

where A is a 2x2 matrix and B is a 2x1 vector. The sampling rate is 10 Hz. You may use either the approximate numerical method (do not use more than three terms) or the Laplace transform method. The candidate needs to find the values for the matrices A and B.

4. (20 marks)

The equation for a dynamic system given in discrete time is, (sampling period is T=0.1 seconds).

$$\bar{x}(k+1) = \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.81 & 1.8 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

Design a full state feedback control law of the form,

$$u(k) = -K\bar{x}(k)$$

such that the controlled system is critically damped (i.e. the damping ratio is one), and the undamped natural frequency is 4.0 rad/s. (Hint: Convert the continuous time pole specifications into the z-domain.)

5. (20 marks)

Design a full order observer for the system,

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & -0.81 \\ 1 & 1.8 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$
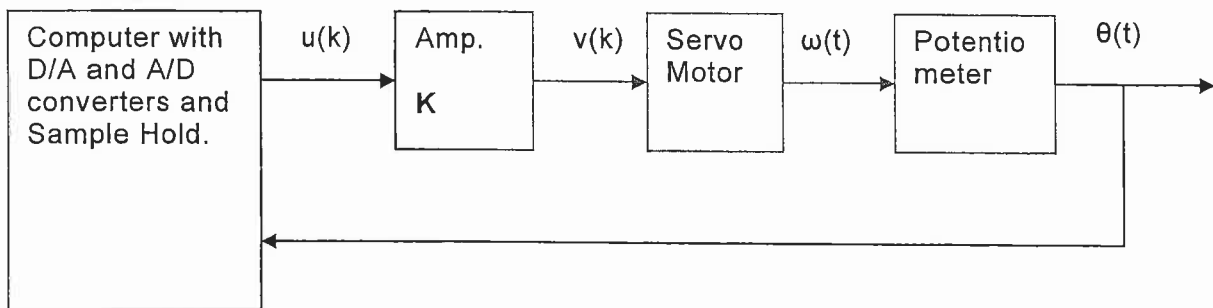
where the output is,

$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

It is specified that the desired characteristic equation of the observer error dynamics is to be $z^2 - z + 0.25 = 0$.

6. (20 marks)

You are given the following system to control with a digital computer. You are to design a position controller for the servo shown below.

| Computer with D/A and A/D converters and Sample Hold. | → u(k) → | Amp. K | → v(k) → | Servo Motor | → ω(t) → | Potentio meter | → θ(t) → |

Where, u(k) is the output of the computer's D/A converter and sample hold. The voltage v(k) is the output of the amplifier (i.e. v(k) = Ku(k)). The angular velocity of the servo motor is ω(t) and the output of the potentiometer is the angular position given by θ(t). You have a model of the servo motor given by the diagram,
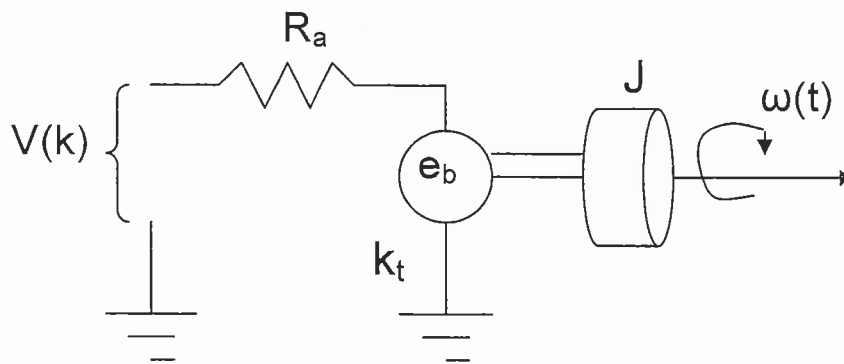
Where k_t is the motor torque constant in N-m/Amp,, J is the polar moment of inertia, R is the armature resistance. You are told that the motor must go to its commanded position within ½ sec and hold that position with a 0.5 degree accuracy. We know the following values, k_t= 1.0 N-m/Amp, J = 0.1 kg·m² and R = 10 Ω. Discuss your design consideration with regard to the following issues,

1) Design Specifications.
2) Closed loop and open loop pole locations.
3) Sampling time.
4) Resolution of D/A and A/D converters.
5) Control strategy, will proportional position feedback control work? Why?

Justify your answers based on numerical calculations and assumptions.