NATIONAL EXAMS MAY 2009
07-Elec-B2 Advanced Control Systems

3 hours duration

NOTES:

1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of two calculators, a Casio or a Sharpe.

3. This is a closed-book examination. Tables of Laplace and z-transforms are attached as page 4 and page 5.

4. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

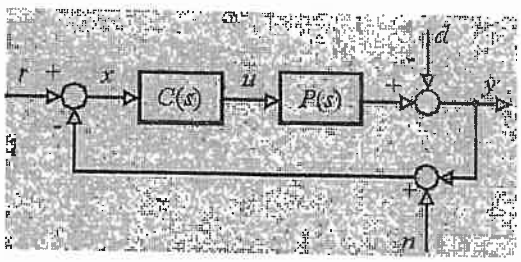
5. All questions are of equal value.

1. Consider the feedback system below with, $C(s) = \frac{sK_p + K_i}{s}$, $P(s) = \frac{2}{s(s+4)}$

(a) Let $K_i = 0$. Find a value for K_p, say $K_p = K_{p0}$, such that the overshoot at $y(t)$ is 15% when there is a step change at $r(t)$. Assume $n(t) = d(t) = 0$.

(b) Let $K_p = K_{p0}$. Find $K_{i\,max}$, the maximum value of K_i for closed loop stability. For $K_i = K_{i\,max}$ determine the closed loop poles.

(c) Let $K_i = K_{i\,max}/2$ and $K_p = K_{p0}$. Let $e(t) = r(t) - y(t)$. Determine the steady state value of $e(t)$ when $r(t)$ = a ramp with slope 2, $d(t) = 0$, and $n(t)$ = a unit step.



2. Consider the open loop dynamics of a satellite attitude control system,

$$\ddot{\phi}(t) + 0.8\dot{\psi}(t) + 0.2\phi(t) = u(t)$$
$$\ddot{\psi}(t) - 0.8\dot{\phi}(t) + 0.2\psi(t) = 0$$

The state vector is given by, $x(t) = \left[\phi(t) \quad \dot{\phi}(t) \quad \psi(t) \quad \dot{\psi}(t)\right]^T$, the control input by, $u(t)$, and the output by, $\phi(t)$.

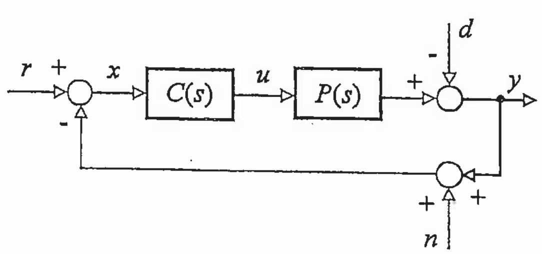
(a) Determine a state space model for the open loop system.

(b) Determine whether the system is stable or not. Justify your answer.

(c) Assuming all of the states are available for feedback, specify a state feedback controller, if it exists, such that the closed loop poles are all located at $s = -1$.

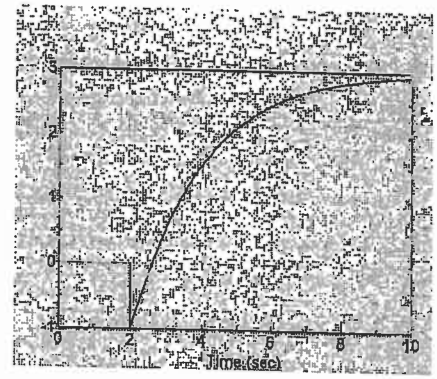
3. Consider the feedback system below with $P(s) = \frac{4}{(s-1)(s+4)}$.

Determine a *proper* and stable $C(s)$ such that the transfer function that relates d to y is given by, $\frac{n(s)}{(s+2)^3}$, $n(s) = b_3s^3 + b_2s^2 + b_1s + b_0$, where the coefficients, b_i, are to be selected as part of the solution. Recall that $C(s)$ is *proper* if the degree of the numerator is less than or equal to that of the denominator.



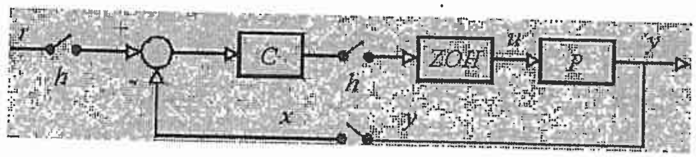
4. Determine the transfer functions, $P(s)$ and $G(s)$ below.

(a) A unit step is applied at the input of an open loop plant, $P(s)$, at time $t = 0$. The measured response is shown on the right. Determine the transfer function, $P(s)$.



(b) When a step of magnitude 2 is applied to the input of a plant, $G(s)$, the steady state output is 10. When a sinusoid of amplitude 2 and frequency 8 rad/sec is applied, the phase lag at the output is 90° and the output amplitude is 15. Assume the system is second order system and has no finite zeros. Find the transfer function, $G(s)$.

5 Consider the system to the right. The input to the ZOH and (continuous) output, y, are uniformly sampled with a sample period of h with $C(z)$ and $P(s)$ given by,



$$C(z) = K, \quad P(s) = \frac{e^{-sh}}{s}$$

(a) Determine the discrete closed loop transfer function, $T(z)$, that relates $X(z)$ to $R(z)$.

(b) Sketch and annotate the root locus as K varies from zero to infinity.

(c) Is the closed loop system stable for all values of K? If not determine the limiting value of K for stability.

(d) Assume r is initially zero up until $t = 0$, and all initial conditions are zero. Suddenly r changes as indicated below.

$$r(0) = 1,\ r(h) = 1,\ r(2h) = 1,\ r(3h) = 1,\ r(4h) = 1$$

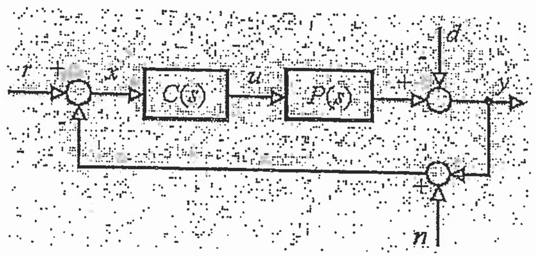
Sketch and carefully annotate the transient response at $y(t)$ for $0 \le t \le 4h$.

6. Consider the feedback system below with, $C(s) = \frac{K}{s}$, $P(s) = \frac{e^{-s}}{s+1}$.

(a) Determine the gain and phase margin when $K = 1$.

(b) Determine the value of K that results in a phase margin of 50 degrees.

(c) Using the value of K from Part (b), determine the steady state value of x when d is a unit step, r is a unit ramp and $n = 0$.



Inverse Laplace Transforms	
$F(s)$	$f(t)$
$\frac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\frac{C+jD}{s+\alpha+j\beta}+\frac{C-jD}{s+\alpha-j\beta}$	$2e^{-\alpha t}\left(C\cos\beta t+D\sin\beta t\right)$
$\frac{A}{(s+\alpha)^{n+1}}$	$\frac{At^{n}e^{-\alpha t}}{n!}$
$\frac{C+jD}{\left(s+\alpha+j\beta\right)^{n+1}}+\frac{C-jD}{\left(s+\alpha-j\beta\right)^{n+1}}$	$\frac{2t^{n}e^{-\alpha t}}{n!}\left(C\cos\beta t+D\sin\beta t\right)$

Inverse z-Transforms	
$F(z)$	$f(nT)$
$\frac{Kz}{z-a}$	Ka^{n}
$\frac{(C+jD)z}{z-re^{j\varphi}}+\frac{(C-jD)z}{z-re^{-j\varphi}}$	$2r^{n}\left(C\cos n\varphi-D\sin n\varphi\right)$
$\frac{Kz}{(z-a)^{r}}$, $r=2,3\ldots$	$\frac{Kn(n-1)\ldots(n-r+2)}{(r-1)!a^{r-1}}a^{n}$

Table of Laplace and z-Transforms (h denotes the sample period)		
$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{\alpha h}}$
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos\beta h)}{z^2-2z\cos\beta h+1}$
$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin\beta h}{z^2-2z\cos\beta h+1}$
$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha h}\cos\beta h)}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha h}\sin\beta h}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$t\,f(t)$	$-\frac{dF(s)}{ds}$	$-zh\frac{dF(z)}{dz}$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$	$F(ze^{\alpha h})$