Association of Professional Engineers of Ontario

National Examinations May 2009

07-Elec-A3 Signals and Communications

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumption made.
2. This is a Closed-Book exam – no aids other than a calculator are permitted.
3. There are six questions in total, and any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

1. (a) The discrete-time Fourier transform of a signal $x[n]$ is given by

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}.$$

Show that $X(e^{j\Omega})$ is periodic in Ω, regardless of the form of $x[n]$, and state its period.

(b) Let $x[n] = \delta[n] + \delta[n-1]$, where $\delta[n] = 1$ at $n = 0$, and 0 otherwise. Find its DTFT, $X(e^{j\Omega})$.

(c) Use Parseval's relation and the result of part (b) to find the energy of the signal $x[n]$.

2. (a) Show that the Fourier transform $X(j\omega)$ of the signal

$$x(t) = \begin{cases} 5 & -\frac{T}{2} < t \leq \frac{T}{2} \\ 0 & \text{elsewhere} \end{cases}$$

is real-valued, and sketch $X(j\omega)$ as a function of ω (with T as a parameter).

(b) Sketch the signal

$$y(t) = \begin{cases} 5 & -2 < t \leq 0 \\ 7.5 & 0 < t \leq 2 \\ 2.5 & 2 < t \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

(c) For $x(t)$ and $y(t)$ defined above, we can write $y(t) = ax(t) + bx(t-2)$ with $T = 4$. Identify the constants a and b, and hence find $Y(j\omega)$, the Fourier transform of $y(t)$.

3. A discrete-time filter is described by the difference equation

$$y[n] = x[n] + 2x[n-1] + 0.5y[n+1]$$

and is initially at rest.

(a) Find its transfer function $H(z)$, and hence its poles and zeros.

(b) If $x[0] = 1, x[1] = 0.5, x[2] = -1.5$ and all other $x[n]$ values are zero, write down five values of $y[n]$, the output of the above LTI system. (Note that the system is *non-causal.*)

(c) If the input is $x[0] = -0.5, x[1] = 1$, and all other $x[n]$ values are zero, find the output of the LTI system.

4. (a) Consider the pulse train described by

$$p(t) = \begin{cases} 10 & -0.05 + k < t < 0.05 + k \\ 0 & \text{elsewhere} \end{cases}$$

where k is any integer, which has a period of 1 second, and a duty cycle of 0.1. Find $P(j\omega)$, the Fourier transform of $p(t)$. (*Hint:* It is a train of modulated impulses.)

(b) The spectrum of a signal $x(t)$ is

$$X(j\omega) = \begin{cases} 1 & -\frac{\pi}{2} < \omega < \frac{\pi}{2} \\ 0 & \text{elsewhere} \end{cases}$$

and it is multiplied by $p(t)$ from part (a) to give $y(t) = x(t)p(t)$. Can $x(t)$ be perfectly recovered from $y(t)$? If so, describe briefly how.

5. A continuous-time bandpass filter has the following frequency response:

$$H(j\omega) = \begin{cases} 1 & 90\pi < |\omega| \leq 110\pi \\ 0 & \text{elsewhere} \end{cases}$$

where ω is measured in radians per second.

   (a) The periodic signal $x(t)$, with fundamental period 0.02 seconds, and one cycle described by $x(t) = +1$, for $0 < t \leq 0.01$ and $x(t) = -1$, for $0.01 < t \leq 0.02$, is input to the bandpass filter. Find the corresponding output.

   (b) A message signal $s(t)$ is band-limited so that $S(j\omega) = 0$ when $|\omega| > 10\pi$ rad/s. It is to be transmitted using double sideband suppressed carrier (DSB-SC) modulation at a carrier frequency of 50 Hz. Discuss how you would process the product signal $s(t)x(t)$, with $x(t)$ defined in part (a), in order to create the signal for transmission.

6. Suppose you were designing a digital imaging system, which transmits full-colour images at a rate of 10 frames per second from the camera to the output device (say a monitor). Each frame consists of 1024×768 pixels, each pixel is represented by the three primary colours – red, green and blue – and the intensity of each colour is quantized to 256 levels.

   (a) Find the minimum bit rate that the communication channel from the camera to the output device needs to be able to support.

   (b) If 8-level baseband pulse-amplitude modulation (PAM) is used, find the minimum physical bandwidth that the channel must have.

   (c) As an alternative to the full-colour system, you are also thinking of offering a monochrome (or grayscale) version that uses the same transmission bandwidth in bits/second. How many more pixels can you accommodate in the monochrome system compared to the colour one, keeping the number of quantization levels the same? Or, keeping the same number of pixels, how many quantization levels can you now offer?