

National Exams May 2009

04-BS-6: Mechanics of Materials

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.
2. Candidates may use one of two calculators, the Casio or Sharp approved models.

This is a Closed Book exam. However candidates are permitted to bring the following into the examination room:

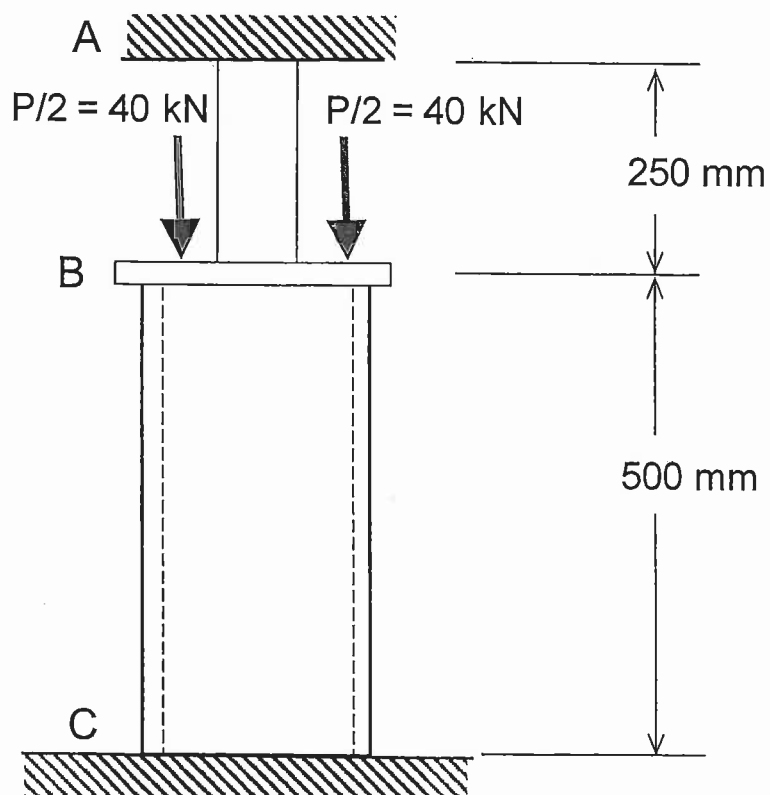
- ONE aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae.
Example problems and solutions to problems are not allowed!
3. Any five questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
 4. All questions are of equal value.

NOTE: The aid sheet must be handed in with the exam!

Your exam will not be marked if you do not hand in an aid sheet, unless there is a signed statement by the exam invigilator stating that no aid sheet was used for the exam.

Question 1: A rigid plate at B is used to transfer a load $P = 80 \text{ kN}$ to a steel rod AB ($E = 210 \text{ GPa}$ and $\sigma_y = 350 \text{ MPa}$) and pipe BC made of an aluminum alloy ($E = 70 \text{ GPa}$ and $\sigma_y = 300 \text{ MPa}$). The steel rod AB has a cross-sectional area of 500 mm^2 and the aluminum pipe BC has a cross-sectional area of 2000 mm^2 . The thickness of the rigid plate may be neglected.

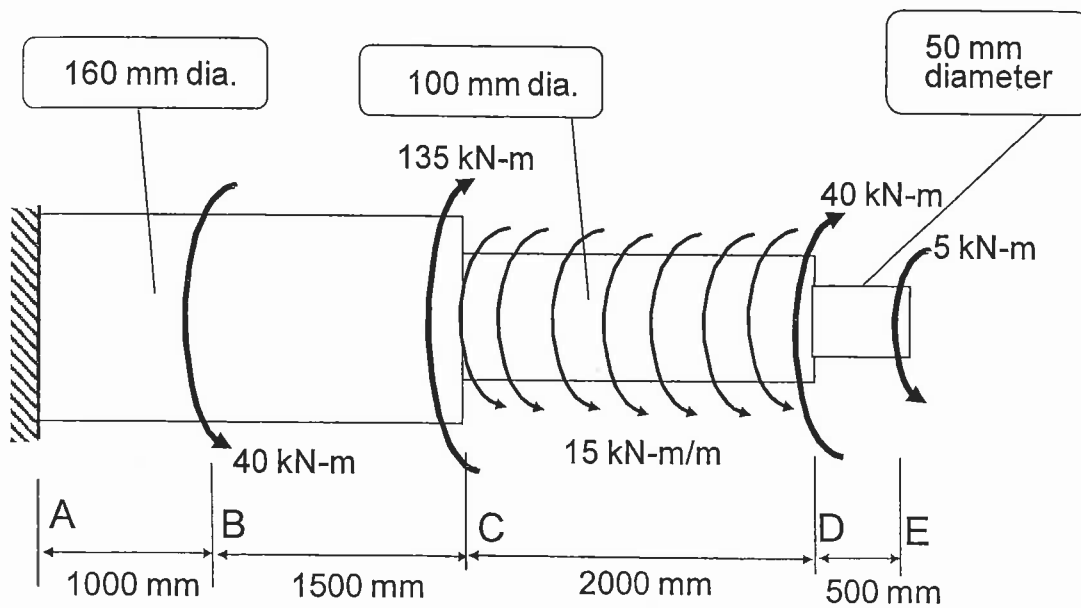
- Determine the forces carried by the steel rod AB and aluminum pipe BC.
- Determine the displacement of the plate at B.



Question 2: A stepped steel shaft with $G = 80 \text{ GPa}$ and $\tau_y = 250 \text{ MPa}$ is subjected to the torques shown (note that one of the torques is a distributed load). Dimensions (length and diameter) are also given.
 [20 marks]

[12 marks] (a) Determine the maximum shear stress in the shaft, and sketch the corresponding variation of shear stress along the shaft radius at this location.

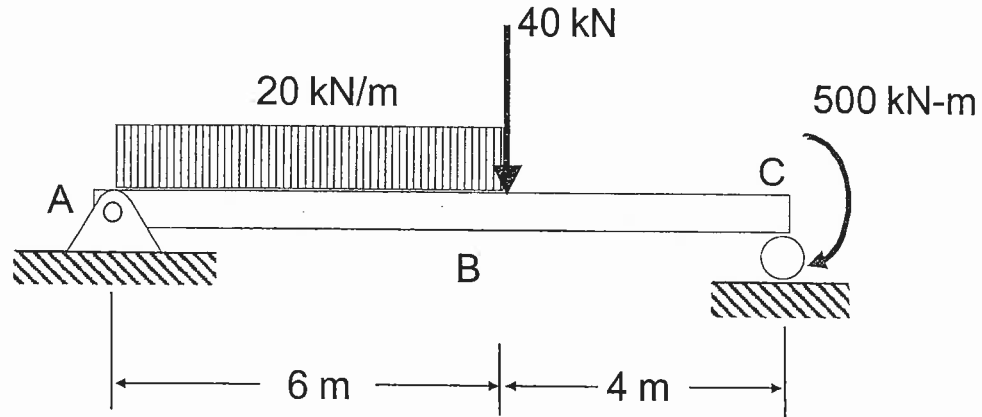
[8 marks] (b) Determine the rotation (in degrees) at the end of the shaft.



Question 3: A simply supported beam supports the loads shown below. The beam is a wide flange W 530 x 82 cross-section using steel with an allowable normal stress of 260 MPa and allowable shear stress of 75 MPa. The elastic modulus of the steel is 200 GPa.

[12 marks] (a) Determine the shear and moment throughout the beam as functions of x . This means that you need to give formula(s) for $V(x)$ and $M(x)$ along the length of the beam.

[8 marks] (b) Next construct the shear force and bending moment diagrams. Remember to label points of maximum and negative bending moment, as well as any inflection points. Show your work by indicating exactly how you obtained your answer.



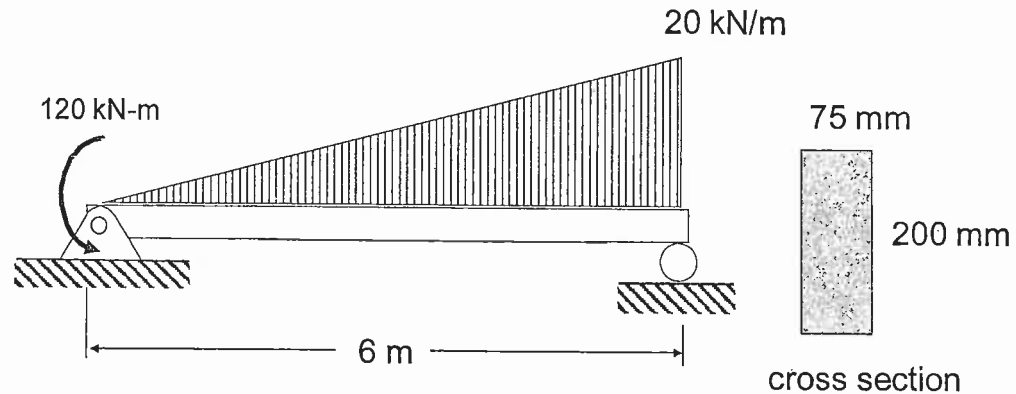
Remember that it is important you set this problem up correctly by calculating the correct reaction forces at the support(s).

No credit will be given for a solution using the principle of superposition, when combinations of existing solutions are used to find an answer.

Question 4: A simply supported beam supports a triangularly distributed load (with a maximum intensity of 20 kN/m) and concentrated couple at the end of the beam of 120 kN-m (see below). The beam is rectangular in cross-section with a 75 mm width and 200 mm depth. The beam is made from steel having an allowable normal stress of 240 MPa and elastic modulus of 200 GPa.

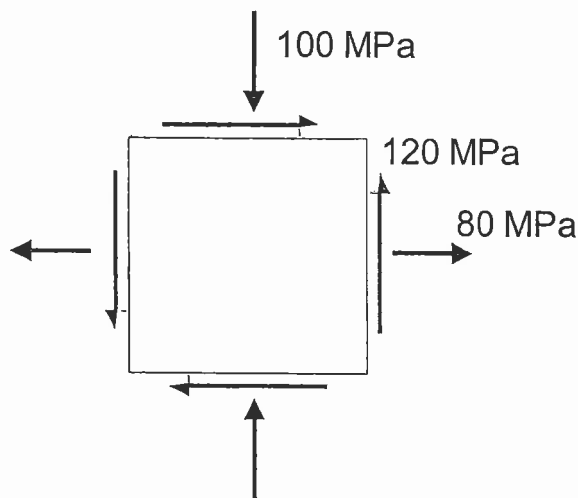
[18 marks] (a) determine the maximum deflection in the beam using the method of integration.

[2 marks] (b) recognizing that computation of deflection is a lengthy process using the method of integration, explain how you would compute the deflection of this beam in a design situation.



Question 5: For an element in a state of plane stress subjected to the normal and shear stresses shown below, use the Mohr's circle solution (*not* the transformation equations) to determine the following:
[20 marks]

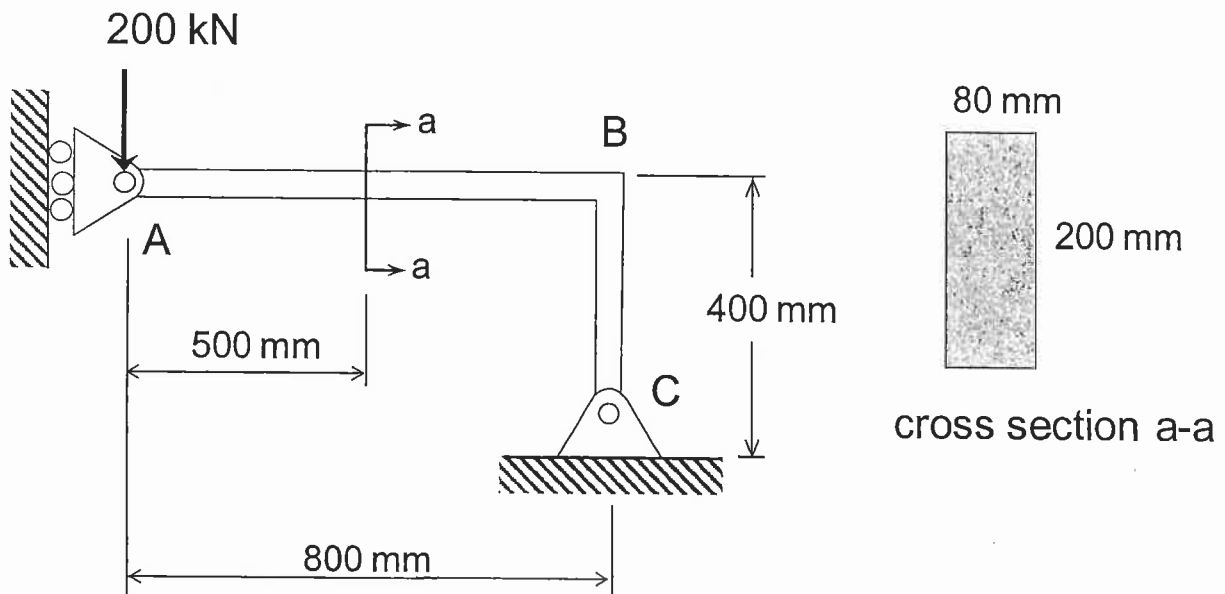
- the principal stresses and orientation of the principal planes, showing your answer on a properly oriented element.
- the maximum in-plane shear stress (and associated normal stresses) and orientation of the corresponding planes. Once again, show your answer on a sketch of a properly oriented element.
- what is the reason for calculating the principal stresses and/or maximum shear stress (not more than 10 words of explanation please).



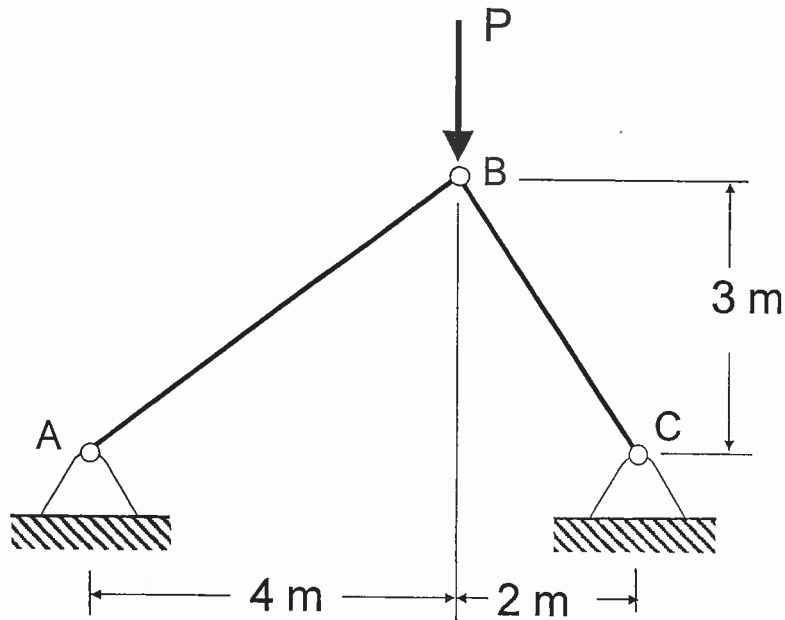
Note that credit will **only** be given for a **solution using Mohr's circle**. This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your **calculations** must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off! The stress transformation equations can only be used to check your answer.

Question 6: A vertical force of 200 kN is applied to the L-shaped element with a rectangular cross-section as shown. The element is supported by a fixed pin at C and by a roller at A. [20 marks]

Determine the distribution of normal and shear stresses in the rectangular section at section a-a which is located 500 mm out from the support at A. Make sure to show your answers on a sketch.



Question 7: Determine the largest load P that can be applied to the frame structure below given [20 marks] that members AB and BC are made of 50 mm diameter steel rods with a yield strength of 300 MPa and elastic modulus of 200 GPa. Use a safety factor of 2 against buckling and consider buckling in the plane of the structure only.



Note: $A_{\text{circle}} = \pi r^2$ and $I_{\text{circle}} = \pi r^4 / 4$

Question 8: The composite member below is made of a rectangular wood section (200 mm by 250 mm in cross-section) reinforced with two 10 mm thick steel plates as shown. The beam supports a maximum positive moment of 60 kN-m in bending about the horizontal axis. The wood has an allowable normal stress of 8 MPa and elastic modulus of 10 GPa, while the steel has an allowable normal stress of 240 MPa and elastic modulus of 200 GPa.

[18 marks] Determine the maximum stress in the wood and steel.

[2 marks] Explain whether the steel plates need to be bonded to the wood section to have composite behaviour and give reasons for your answer.

