Professional Engineers Ontario

National Exams - December, 2009
98-Civ-B9

Applications of the Finite Element Method

3 hours duration

Notes:

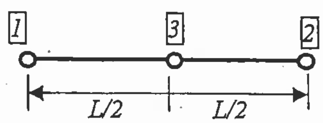
1. There are 3 pages in this examination.
2. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. This is a closed book examination, with two 8½×11 in^2 pages of hand written notes.
4. One of two calculators is permitted any Casio or Sharp approved models
5. **Attempt to answer all three problems.**
6. All problems are of equal value.

Problem 1:

1. Explain why the stiffness matrix of a 2D solid element is symmetric?
2. Explain why the global stiffness matrix generated by the finite element method is singular prior to the application of any boundary conditions?
3. Select the right answer to the following statement;
   "The strain energy of an elastic structure calculated by the finite element method is:"

   a. higher than the exact value
   b. equal to the exact value
   c. lower than the exact value

   and explain why?
4. Explain why the shape functions needed for Bernoulli's beam formulation should be a polynomial of degree 3.
5. Draw the approximate shape functions for the truss element shown in the following figure.



Element 1

6. Explain why the element stresses across element boundaries are, in general, not in equilibrium?
7. When is it possible to adopt the assumption of plane strain for the analysis of a three-dimensional structure?
8. Why are linear quadrilateral elements not good candidates for flexural dominant two-dimensional elasticity problems?
9. Explain how do you proceed to select the necessary required modes for a dynamic analysis of a structure in the frequency domain?
10. Explain how to select the time step for a dynamic analysis of a structure in the time domain.

Problem 2

For the plane strain element shown in Figure 2, determine the element stresses σ_x, σ_x, and τ_{xy} corresponding to the following nodal displacements:

$$u_1 = 5.0\,mm \quad v_1 = 2.0\ mm$$
$$u_2 = 0.0\,mm \quad v_2 = 0.0\ mm$$
$$u_3 = 5.0\,mm \quad v_3 = 0.0\ mm$$

Use $E = 70\,GPa$ and $\nu = 0.3$.

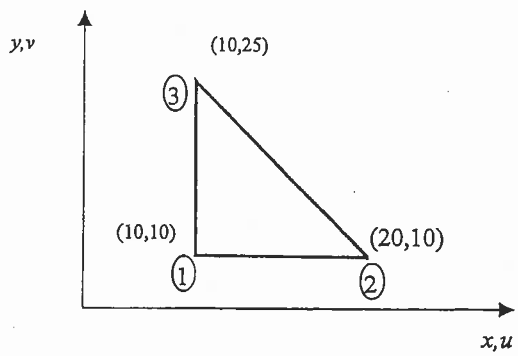


Figure 2 (all dimensions are in millimetres).

Problem 3

Using the finite element method, calculate the reactions and draw the shear and moment diagrams for the structure shown below (Figure 3). In addition to the illustrated loading, support 2 is subjected to a settlement of 12 mm downward. All members have the same rigidity $EI = 700\ 00\ kN \cdot m^2$. The stiffness matrix of a beam element is given by:

$$[k] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

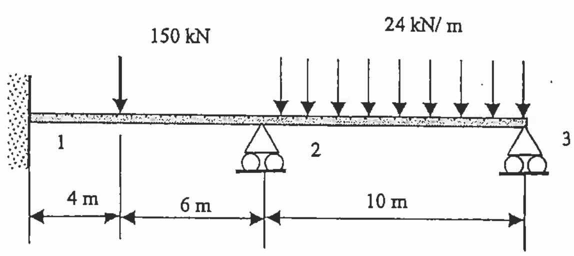


Figure 3