

PROFESSIONAL ENGINEERS OF ONTARIO

ANNUAL EXAMINATIONS – December 2009

07-Mec-B10 Finite Element Analysis

3 hours duration

INSTRUCTIONS:

1. **If doubt exists as to the interpretation of any of the questions, the candidate is urged to submit a clear statement of the assumption(s) that he/she has had made with the answer.**
2. **The examination paper is open book and so candidates are permitted to make use of any textbooks, references or notes that they wish to use.**
3. **Any non-communicating calculator is permitted. A calculator that can handle small matrices will speed the solving of the problems. Candidates must indicate the type of calculator(s) that they have used by writing the name and model designation of the calculator(s) on the first inside left hand sheet of the first examination workbook.**
4. **Candidates are required to attempt five questions. Solve all problems using finite element method.**
5. **All questions carry the same value. Indicate which five questions are to be marked on the cover of the first examination workbook.**

PROBLEM 1 (20 POINTS)

Consider the following differential equation:

$$\frac{d^2 T}{dx^2} = 2x - 1 \quad \text{for } 0 \leq x \leq 1$$

subject to boundary conditions:

$$T(0) = 0, \quad \left. \frac{dT}{dx} \right|_{x=1} = 5$$

a. Using the trial function:

$$\phi_1 = x^1$$

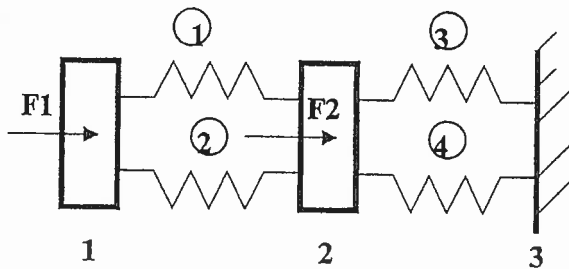
determine the coefficient matrix and right hand side vector of the general N- parameter Rayleigh-Ritz approximation.

b. Calculate the two parameter (N=2) Ritz approximation.

PROBLEM 2 (20 POINTS)

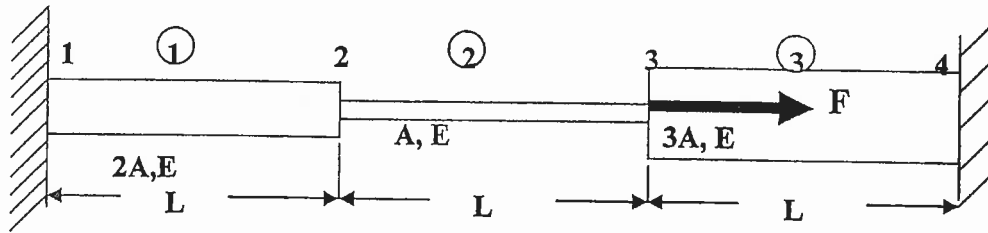
Using the one-dimensional finite elements formulation, develop the structure stiffness matrix and apply the boundary conditions for the assembly shown below.

If the spring constants for the structure shown below are $k_1 = 3000 \text{ N/m}$, $k_2 = k_3 = 5000 \text{ N/m}$ and $k_4 = 10,000 \text{ N/m}$ with the loads $F_1 = 1500 \text{ N}$ and $F_2 = 1000 \text{ N}$ find the displacements of nodes 1 and 2 and the forces in each spring.



PROBLEM 3 (20 POINTS)

Assemble the total stiffness matrix K and the load and displacements equations for the truss assemble shown below. State clear all your assumptions. Do not solve.



PROBLEM 4 (20 POINTS)

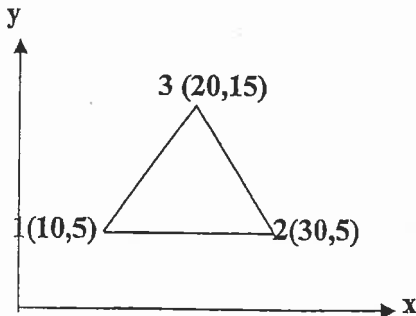
For the plane strain element shown below, the nodal displacements are given as:

$$u_1 = 0.005 \text{ mm} \quad u_2 = 0.000 \text{ mm} \quad u_3 = 0.005 \text{ mm}$$

$$v_1 = 0.002 \text{ mm} \quad v_2 = 0.000 \text{ mm} \quad v_3 = 0.000 \text{ mm}$$

Determine the element stresses $\sigma_x, \sigma_y, \tau_{xy}$, the principal stresses σ_1 and σ_2 , and the principal angle θ_p .

The plate thickness is 1 mm; $E = 210 \text{ MPa}$ and $\nu = 0.3$. All coordinates are millimetres.

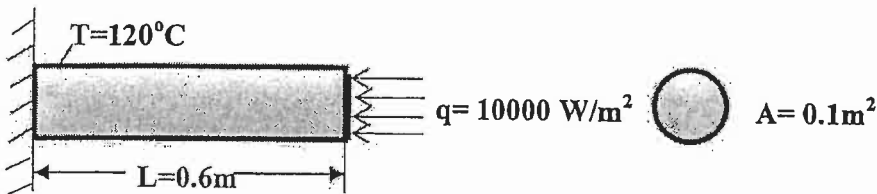


PROBLEM 5 (20 POINTS)

A fin shown below is insulated on the perimeter. The left end has a constant temperature of 120°C . A heat flux of 10000 W/m^2 acts on the right end of the fin. Thermal conductivity is $k=10\text{ W/m}^{\circ}\text{C}$, the area of the fin is 0.1 m^2 .

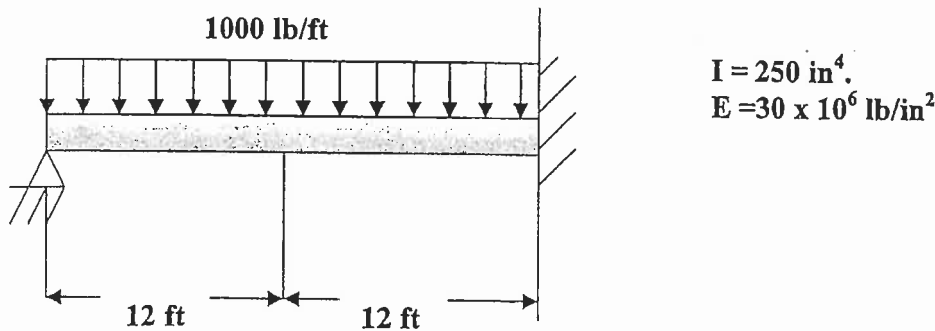
Determine the temperature at $L/4$, $L/2$, $3L/4$ and L , where $L=0.6\text{m}$

Formulate the problem, using finite elements analysis.



PROBLEM 6 (20 POINTS)

For the beam under the loading shown in determine the nodal displacements and the slope at midpoint.



PROBLEM 7 (20 POINTS)

PART A. 10 points

Explain in a sentence or two the following concepts:

- skyline solution
- symmetric banded matrix
- Gauss elimination
- to what does the term *degree of freedom* refer?

PART B. 10 points

b. Sketch the frame of a bicycle and try to indicate how you will model it using frames and solving it using finite elements. Indicate the loads and boundary conditions.