

Association of Professional Engineers of Ontario

National Examinations December 2009

07-Elec-A3 Signals and Communications

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumption made.
2. This is a Closed-Book exam – no aids other than a calculator Casio or Sharp approved models
3. There are six questions in total, and any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

1. Determine the average power and the rms value of:

(a) $g(t) = C \cos(\omega_0 t + \theta)$.

(b) $g(t) = C_1 \cos(\omega_1 t + \theta_1) + C_2 \cos(\omega_2 t + \theta_2)$ where $\omega_1 \neq \omega_2$.

(c) $g(t) = D e^{j\omega_0 t}$, where D is complex.

(d) The periodic signal with fundamental period $T = 1$, given by

$$g(t) = \begin{cases} 1, & 0 < t \leq 0.5 \\ -0.5, & 0.5 < t \leq 1 \end{cases}$$

within one period.

2. (a) Find the time autocorrelation function of the signal $g(t) = e^{-at}u(t)$, and from it determine the energy spectral density (ESD) of $g(t)$.
- (b) Using the ESD found in part (a), estimate the essential bandwidth W rad/s of the signal $g(t)$ if the essential band is required to contain 95 percent of the signal energy. You may need the integral

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right).$$

3. A discrete-time filter is described by the difference equation

$$y[n] = x[n] + 2x[n - 1] + 0.5y[n - 2]$$

and is initially at rest.

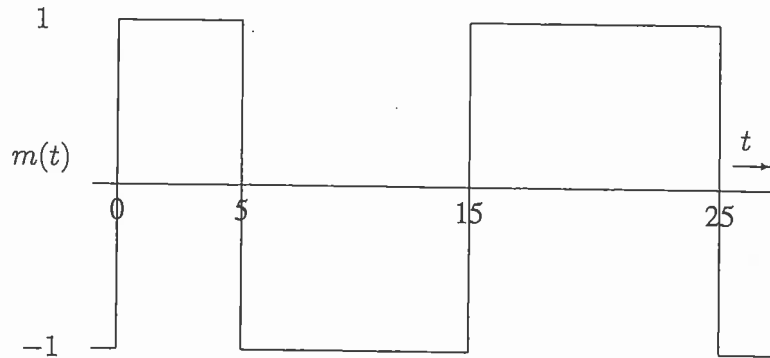
- (a) If the input is $X(z) = 1 + \sqrt{0.5}z^{-1}$, find the output in the time domain, $y[n]$.
- (b) Find the poles and zeros of the filter, and comment on the stability of the filter.
- (c) Find expressions for the magnitude response and the phase response of this filter.

4. Consider a composite wave obtained by adding a noncoherent carrier $A_c \cos(2\pi f_c t + \phi)$ to a DSB-SC wave $\cos(2\pi f_c t)m(t)$, where $m(t)$ is the band-limited message signal, with a bandwidth very much smaller than f_c . This composite wave is applied to an ideal envelope detector. Describe how you would recover $m(t)$, if possible, from the envelope detector output in the following cases:

- (a) $\phi = 0$.
- (b) $\phi = 90$ degrees.
- (c) $\phi \neq 0$ or 90 degrees.

(In all cases, ϕ is known to the receiver.)

5. Consider the modulating (message) signal $m(t)$ sketched below:



The unit of time is milliseconds in this figure. The message $m(t)$ is to be frequency modulated (FM), with a transmitted signal given by

$$\phi_{FM}(t) = A \cos \left[2\pi f_c t + k_f \int_{-\infty}^t m(u) du \right],$$

where $k_f = 2\pi \times 10^5$ and $f_c = 100$ MHz. (Assume that $m(t)$ is zero mean over a long period of time.)

- Sketch the signal $\phi_{FM}(t)$.
 - Assuming the availability of an ideal differentiator, describe the demodulator for an FM signal, including block diagrams and equations explaining how $m(t)$ is recovered from $\phi_{FM}(t)$.
6. A signal $g(t) = \text{sinc}^2(5\pi t)$ is sampled using a train of uniformly spaced impulses at a rate of (i) 5 Hz, (ii) 10 Hz and (iii) 20 Hz. For each of the three cases,
- Sketch the sampled signal.
 - Sketch the spectrum of the sampled signal.
 - Explain whether you can recover the signal $g(t)$ from the sampled signal, and if so, how.