

NATIONAL EXAMINATIONS DECEMBER 2009

04-BS-5 ADVANCED MATHEMATICS

3 Hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumption made.
2. Candidates may use one of the approved Casio or Sharp calculators. This is a Closed Book Exam. However, candidates are permitted to bring **ONE** aid sheet (8.5"x11") written on both sides.
3. Any five (5) questions constitute a complete paper. Only the first five answers as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme

1. 20 marks
2. 20 marks
3. 20 marks
4. (A) 10 marks ; (B) 10 marks
5. 20 marks
6. (a) 12 marks ; (b) 8 marks
7. (a) 10 marks ; (b) 10 marks

1. Consider the following differential equation

$$(x^2 + 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + 2xy = 0$$

Find two linearly independent power series solutions about the ordinary point  $x=0$  subject to the initial conditions  $y(0)=2$  and  $y'(0)=3$ .

2. Find the Fourier series expansion of the periodic function  $F(x)$  of period  $p=2\pi$ .

$$F(x) = \begin{cases} x(\pi + x) & -\pi < x \leq 0 \\ x(\pi - x) & 0 < x \leq \pi \end{cases}$$

3. Find the Fourier integral representation of the following function

$$f(x) = \begin{cases} \frac{\pi}{2} \cos x & \text{if } |x| < \frac{\pi}{2} \\ 0 & \text{if } |x| > \frac{\pi}{2} \end{cases}$$

Hint: 
$$f(x) = \int_0^{\infty} [A(w) \cos wx + B(w) \sin wx] dw$$

where 
$$A(w) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(v) \cos wv dv \quad ; \quad B(w) = \int_{-\infty}^{\infty} f(v) \sin wv dv$$

4(A). Given the following data find Newton's interpolating polynomial of highest possible degree.

x	0	1	2	3	4
f(x)	4	9	15	18	38

4(B). Find the approximate value of the derivative of the function  $f(x)$  for the four given values of  $x$ . Use the differentiation formulas supplied below. (Hint: Let  $x_0=0$  and  $h=0.5$ ).

$x$	0	0.5	1.0	1.5
$f(x)$	6	6.75	5	0.75

$$f'(x_0) \approx \frac{1}{6h} [-11f(x_0) + 18f(x_0 + h) - 9f(x_0 + 2h) + 2f(x_0 + 3h)]$$

$$f'(x_0 + h) \approx \frac{1}{6h} [-2f(x_0) - 3f(x_0 + h) + 6f(x_0 + 2h) - f(x_0 + 3h)]$$

$$f'(x_0 + 2h) \approx \frac{1}{6h} [f(x_0) - 6f(x_0 + h) + 3f(x_0 + 2h) + 2f(x_0 + 3h)]$$

$$f'(x_0 + 3h) \approx \frac{1}{6h} [-2f(x_0) + 9f(x_0 + h) - 18f(x_0 + 2h) + 11f(x_0 + 3h)]$$

5. Use the Romberg algorithm with  $n = 2$  to find the area bounded by  $f(x)=4/(4\sin^2x+\cos^2x)$ ,  $x=0$ ,  $x=\pi/2$  and  $y=0$ .

Hint: The Romberg algorithm produces a triangular array of numbers, all of which are numerical estimates of the definite integral  $\int_a^b f(x)dx$ . The array is denoted by the

following notation:

$R(0,0)$   
 $R(1,0)$              $R(1,1)$   
 $R(2,0)$              $R(2,1)$              $R(2,2)$   
 where

$$R(0,0) = \frac{1}{2}(b-a)[f(a) + f(b)]$$

$$R(n,0) = \frac{1}{2} R(n-1,0) + h \sum_{k=1}^{2^{n-1}} f[a + (2k-1)h]$$

$$\text{where } h = \frac{b-a}{2^n}$$

$$R(n,m) = R(n,m-1) + \frac{1}{4^m - 1} [R(n,m-1) - R(n-1,m-1)]$$

6. The equation  $x^3 - 1 - \sin x = 0$  has a root close to  $x_0=1$ .

(a) The given equation can be transformed into the form  $x=g(x)$  in several ways. Find two forms that will enable you to find a better approximation of the root using fixed point iteration five times. (Note: Carry five significant digits in your calculations).

(b) Use the following iterative formula four times to find a better approximation of the root. Start with  $x_0=1$ . (Note: Carry six significant digits in your calculations).

$$x_{i+1} = x_i - \frac{f(x_i)}{f^{(1)}(x_i) - \frac{f(x_i)f^{(2)}(x_i)}{2f^{(1)}(x_i)}}$$

Hint: Let  $f(x) = x^3 - 1 - \sin x$ . Note that  $f^{(1)}(x)$  represents the first derivative of  $f(x)$ . Similarly  $f^{(2)}(x)$  represents the second derivative of  $f(x)$ .

7. The symmetric positive definite matrix  $A = \begin{bmatrix} 9 & -3 & 6 \\ -3 & 17 & -14 \\ 6 & -14 & 17 \end{bmatrix}$  can be written as the

product of a lower triangular matrix  $L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$  and its transpose  $L^T$ , that is

$$A = LL^T.$$

(a) Find  $L$  and  $L^T$ .

(b) Use the results obtained in (a) to solve the following system of three linear equations:

$$9x_1 - 3x_2 + 6x_3 = 6$$

$$-3x_1 + 17x_2 - 14x_3 = 38$$

$$6x_1 - 14x_2 + 17x_3 = -34$$