National Exams

December 2008

07-Elec-B1, Digital Signal Processing

3 hours duration

NOTES:

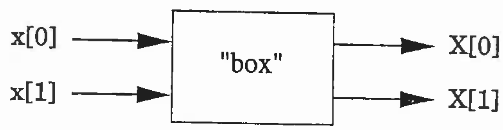
1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is ¡**CLOSED BOOK EXAM** (No notes just one textbook of your choice) plus a simple calculator is permitted.
3. FOUR(4) questions constitute a complete paper. The first four questions as they appear in the answer book will be marked.
4. All questions are of equal value.
5. Clarity and organization of the answer are important.

1. **(25 marks total)** Let $u[n] = \{a, b, c\}$ and $v[n] = \{0, 1\}$ be two finite-length, real-valued sequences.

   (a) (3pts) Determine $z_l[n] = u[n] \circledast v[n]$, the linear convolution of the sequences $u[n]$ and $v[n]$.

   (b) (3pts) Determine $z_c[n] = u[n] ③ v[n]$, the 3-point circular convolution of the sequences $u[n]$ and $v[n]$.

   (c) (5pts) Let $\{x[n]\} = \{x[0], x[1]\}$ be a 2-element sequence and let $\{X[k]\} = \{X[0], X[1]\}$ be the 2-point Discrete Fourier Transform (DFT) of $\{x[n]\}$. Determine the signal flow diagram for a 2-input, 2-output system, the "box", where $\{x[n]\}$ is the input and $\{X[k]\}$ is the output.



   (d) (4pts) Assume that you are ONLY allowed to perform scalar multiplications external to the "box" you determined in part (c). Show how you would use the "box" such that its output is $x[n]$ when $\{X[k]\}$ is the input.

   (e) (5pts) Determine $z_l[n]$ using 2-point DFT and 2-point IDFT operations. (You may want to use what you derived in parts (c) and (d).)

   (f) (5pts) Let $v[n]$ be a length L_v sequence, and let $u[n]$ be a right-hand sequence (a sequence that is non-zero for $n \leq 0$). You are asked to calculate $z_l[n] = u[n] \circledast v[n]$ using "block filtering" (i.e., using either the overlap-add or the overlap-save method) implemented via L-point DFT/IDFT operations. Determine the minimum allowed value of L.

2. **(25 marks total)** Consider the LTI discrete system described by the difference equation

$$y[n] = x[n] + 2x[n-1] + x[n-2] \qquad (1)$$

Let the input to the system be the 4-sample sequence

$$x[n] = \{ \underset{\underset{n=0}{\uparrow}}{1}, \quad 0, \quad 1, \quad 1\},$$

(a) (2pts) Determine the impulse response sequence $h[n]$ which describes the above system.

(b) (2pts) Determine the system output $y[n]$ when $x[n]$ is the input using the difference equation given in Equation (1).

(c) (2pts) Determine $y[n]$ using linear convolution.

(d) (3pts) Determine $y[n]$ using circular convolution (implemented in time domain).

(e) (5pts) Determine $y[n]$ using DTFT/IDTFT.

(f) (7pts) Determine $y[n]$ using DFT/IDFT.

(g) (4pts) $H(e^{j\omega})$ and $\{H_4[k]\}$ be the DTFT and 4-point DFT of $h[n]$, respectively. Determine $\{H_4[k]\}$ directly from $H(e^{j\omega})$.

3. **(25 marks total)** Consider the LTI discrete system with the frequency response function

$$H(e^{j\omega}) = 2\cos\omega + 2e^{j\omega}. \tag{2}$$

(a) (10pts) Plot the magnitude and phase response functions.

(b) (5pts) Determine h[n], the impulse response sequence that describes the system.

(c) (10pts) Let $x[n]$ be the 8-sample sequence described by its 8-point DFT coefficients:

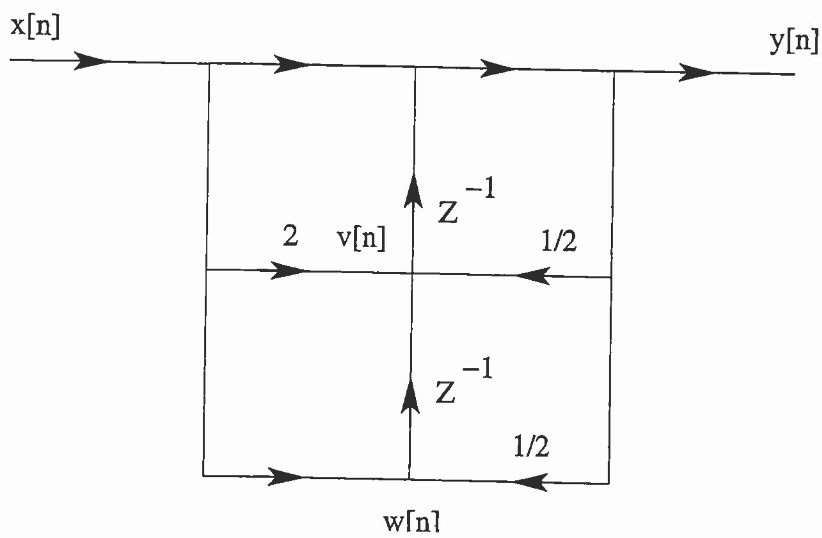
$$X_8[k] = \sum_{n=0}^{7} x[n] W_8^{kn}, k = 0, 1, ..., 7; \tag{3}$$

such that

$$X_8[k] = \{ \underset{\substack{\uparrow \\ k=0}}{0}, \; 2, \; 0, \; 1, \; 0, \; 1, \; 0, \; 2\},$$

Determine the system output $y[n]$ when $x[n]$ is the input.

4. **(25 marks total)** Consider the signal flow graph shown in the below figure.



(a) (10pts) Using the node variables indicated, write the set of difference equations represented by this network.

(b) (10pts) Draw the flow graph of an equivalent system that is the cascade of two first-order systems.

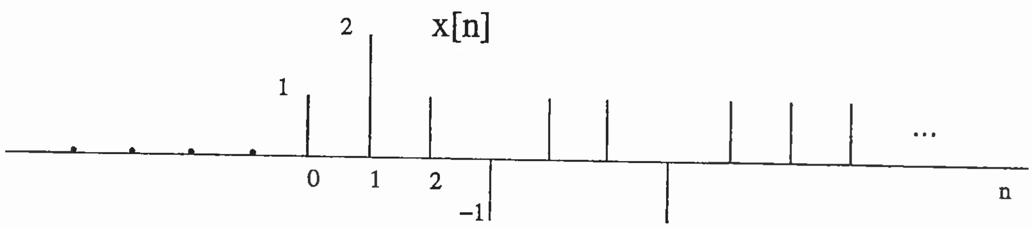
(c) (5pts) Is the system stable? Explain.

5. **(25 marks total)** Consider a linear time-invariant system whose system transfer function $H(z)$ is

$$H(z) = \frac{z^{-2}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})} \tag{4}$$

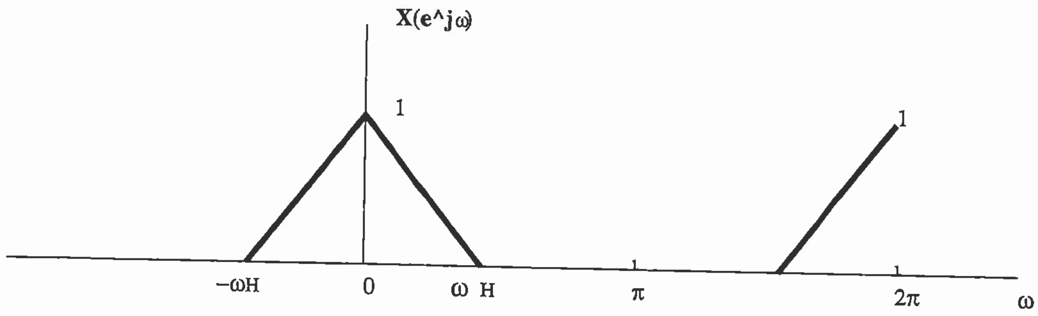
(a) (7pts) Suppose the system is known to be stable. Determine the output $y[n]$ when the input $x[n]$ is the unit step sequence.

(b) (8pts) Suppose the region of convergence of $H(z)$ includes $z = \infty$. Determine $y[n]$ evaluated at $n = 2$ when $x[n]$ is as shown in the below figure.



(c) (10pts) Suppose we wish to recover $x[n]$ from $y[n]$ by processing $y[n]$ with an LTI system whose impulse response is given by $h_i[n]$. Determine $h_i[n]$. Does $h_i[n]$ depend on the region of convergence of $H(z)$?, explain.

6. **(25 marks total)** Consider the sequence $x[n]$ whose Fourier Transform $X(e^{j\omega})$ is shown in the below figure with the bandwidth of $(-\omega_H, \omega_H)$.



Define

$$x_s[n] = \begin{cases} x[n] & n = Mk \quad k = 0, \pm 1, \pm 2, ... \\ 0 & otherwise \end{cases} \qquad (5)$$

and

$$x_d[n] = x_s[Mn] = x[Mn]$$

(a) (15pts) Sketch $X_s(e^{j\omega})$ for each of the following cases:

i. $M = 3$, $\omega_H = \pi/2$

ii. $M = 3$, $\omega_H = \pi/4$

(b) (10pts) What is the maximum value of ω_H that will avoid aliasing when $M = 3$?