Association of Professional Engineers of Ontario

National Examinations December 2008

07-Elec-A3 Signals and Communications

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumption made.
2. This is a Closed-Book exam – no aids other than a calculator are permitted.
3. There are six questions in total, and any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

1. (a) Write down the definition of the continuous-time Fourier series (CTFS) of a periodic signal $x(t)$, with period T.

   (b) If a continuous-time signal $x(t)$ with fundamental frequency f_0 has CTFS coefficients X_k, what is its Fourier transform $X(j\omega)$?

   (c) Let $x(t) = 2\cos(2\pi t/7) + \cos(\pi t)$. Find the CTFS of $x(t)$, and hence its Fourier transform $X(j\omega)$.

2. (a) Show that the Fourier transform of the signal

$$x(t) = \begin{cases} 1 & -\frac{T}{2} < t \leq \frac{T}{2} \\ 0 & \text{elsewhere} \end{cases}$$

   is in the form of a sinc function, defined as $\text{sinc}(\theta) = \frac{\sin \pi\theta}{\pi\theta}$.

   (b) Using the time-frequency duality property of the Fourier transform and the result in part (a), find the Fourier transform of

$$y(t) = \text{sinc}(t) = \frac{\sin \pi t}{\pi t}.$$

3. A discrete-time filter is described by the difference equation

$$y[n] = 10x[n] + 2x[n-1] + 0.8y[n-1]$$

and is initially at rest.

(a) Find its transfer function $H(z)$, and hence its poles and zeros.

(b) Find its impulse response $h[n]$. Is this an IIR or an FIR filter?

(c) Find the transfer function $H^{-1}(z)$ of the inverse of this filter. Is the inverse filter causal and stable? Explain your answer.

(d) Suppose the input to the filter with impulse response $h[n]$ is

$$x[n] = \delta[n] - 0.8\delta[n-1].$$

Find the corresponding filter output. *(Hint: Convolution is not necessary.)*

4. (a) State the Nyquist sampling theorem, and give an example in which violation of the conditions of the theorem results in signal distortion.

(b) A baseband signal with one-sided bandwidth 50 kHz is to be sampled at a rate of 60 kHz. Find

i. the cutoff frequency of the anti-aliasing filter;

ii. the bit rate required to transmit the sampled signal using 4-level pulse amplitude modulation;

iii. the bandwidth efficiency of the system if a passband channel centered at 1 GHz with a one-sided bandwidth of 50 kHz is used.

5. (a) Sketch block diagrams for (i) the transmitter and (ii) the receiver of a digital communication system with bandpass modulation, assuming no error control coding and a binary modulation format.

   (b) A raised cosine filter has the frequency response

$$H(f) = \begin{cases} T & |f| \le \frac{1-\alpha}{2T} \\ T\cos^2\left[\frac{\pi T}{2\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right] & \frac{1-\alpha}{2T} < f \le \frac{1+\alpha}{2T} \\ 0 & |f| > \frac{1+\alpha}{2T} \end{cases}$$

   where T is the symbol interval, and α is known as the roll-off factor. This is a low-pass filter. Find

   i. the 3-dB cutoff frequency;

   ii. the bandwidth of the transition region from passband to stopband;

   iii. the frequency response of a *square-root* raised cosine filter with roll-off factor α.

   (c) Explain the role of a square-root raised cosine filter in a digital communication system.

6. Assume that the continuous-time signal $x(t)$ is band-limited, i.e. it has a Fourier transform that satisfies $X(j\omega) = 0$ for $|\omega| > \omega_M > 0$.

   (a) Define $w(t) = x(t) + \sin(\omega_c t)$. Sketch the spectrum of $w(t)$. Assume any convenient shape for $|X(j\omega)|$ within its passband, and that $\omega_c > \omega_M$.

   (b) Define $v(t) = w^2(t)$. Sketch the spectra of the three components in $v(t)$, and hence sketch the spectrum of $v(t)$.

   (c) The signal $v(t)$ is input to a bandpass filter with frequency response $H(j\omega)$ having cutoff frequencies ω_l and ω_h, where $\omega_l < \omega_h$. The filter's passband gain is unity. Specify all the constraints or requirements on ω_c, ω_M, ω_l and ω_h necessary to ensure that the filter output $y(t) \propto x(t)\sin(\omega_c t)$.

   (d) Assuming the conditions you found in part (c) are satisfied, how would you filter $v(t)$ to recover $x^2(t) + \sin^2(\omega_c t)$?

Marking Scheme

1. (a) 5 marks (b) 8 marks (c) 7 marks
2. (a) 10 marks (b) 10 marks
3. 5 marks in each part.
4. (a) 6 marks (b.i) 4 marks (b.ii) 5 marks (b.iii) 5 marks
5. (a) 6 marks (b.i) 3 marks (b.ii) 3 marks (b.iii) 4 marks (c) 4 marks
6. 5 marks in each part.